# NON-RENEWABLE RESOURCES, EXTRACTION TECHNOLOGY, AND ENDOGENOUS GROWTH*

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**Abstract**

We develop a theory of innovation in non-renewable resource extraction and economic growth. Firms increase their economically extractable reserves of non-renewable resources through investment in new extraction technology and reduce their reserves through extraction. Our model allows us to study the interaction between geology and technological change, and its effects on prices, total output growth, and the resource intensity of the economy. The model accommodates long-term trends in non-renewable resource markets – namely stable prices and exponentially increasing extraction – for which we present data extending back to 1792. The paper suggests that over the long term, increasing consumption of non-renewable resources fosters the development of new extraction technologies and hence offsets the exhaustion of higher quality resource deposits. (JEL codes: O30, O41, Q30)

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# 1 Introduction

This paper contributes to resolving a contradiction between theoretical predictions and empirical evidence regarding non-renewable resources. According to theory, economic growth is not limited by non-renewable resources because of three factors: technological change in the use of resources, substitution of non-renewable resources by capital, and returns to scale. Given these factors, growth models with a non-renewable resource typically predict growth in output, decreased non-renewable resource extraction, and an increase in price (see Groth, 2007; Aghion and Howitt, 1998).

However, it is a well-established fact that these predictions are not in line with the empirical evidence from the historical evolution of production and prices of non-renewable resources. The extraction of non-renewable resources has increased over time, and there is no persistent increase in the real prices of most non-renewable resources over the long run (see Krautkraemer, 1998; Livernois, 2009; Von Hagen, 1989).

To resolve this puzzle, the paper develops a theory of technological change in resource extraction in an endogenous growth model. Our starting point is the seminal paper by Nordhaus (1974), in which he suggests that innovation in extraction technology helps overcome scarcity by turning mineral deposits in the Earth's crust into economically recoverable reserves. Nordhaus also points out that the crustal abundance of non-renewable resources is sufficient to continue consumption for hundreds of thousands of years if there is technological change.

Modeling technological change in resource extraction in a growth model is challenging because it adds a layer of dynamic optimization to the model. We boil down the investment and extraction problem to a static problem, which makes our model both simple enough to solve and rich enough to potentially connect to long-run data.

To our knowledge, our model is the first that allows the study of the interaction between technological change and geology, and its effects on prices, total output growth, and its use in the economy. Learning about these effects is important for making predictions of long-run development of resource prices and for understanding the impact of resource production on aggregate output. For example, distinguishing between increasing and constant resource prices in the long run is key to the results of a number of recent papers on climate economics (Acemoglu et al., 2012; Golosov et al., 2014; Hassler and Sinn, 2012; van der Ploeg and Withagen, 2012).

We add an extractive sector to a standard endogenous growth model of expanding varieties and directed technological change by Romer (1986) and Acemoglu (2002), such that aggregate output is produced from a non-renewable resource and an intermediate good.

Modeling the extractive sector has four components: First, we assume that there is a continuum of deposits of declining grades. The quantity of the non-renewable resource is distributed such that it increases exponentially as the ore grades of deposits decrease, as a local approximation to Ahrens (1953, 1954) fundamental law of geochemistry. Although we recognize that non-renewable resources are ultimately finite in supply, we make the assumption that the underlying resource quantity goes to infinity for all practical economic purposes as the grade of the deposits approaches zero. Without innovation in extraction technology, the extraction cost is assumed to be infinitely high.

Second, we build on Nordhaus (1974) idea that reserves are akin to working capital or inventory of economically extractable resources. Firms can invest in grade specific extraction technology to subsequently convert deposits of lower grades into economically extractable reserves. We assume that R&D investment exhibits decreasing returns in making deposits of lower grades extractable, as historical evidence suggests. Once converted into a reserve, the firm that developed the technology can extract the resource at a fixed operational cost.

Third, new technology diffuses to all other firms. As each new technology is specific to a deposit of a certain grade, it cannot be used to extract resources from deposits of lower grades. However, all firms can build on existing technology when they invest in developing new technology for deposits of lower grades. The idea is that firms can, for example, use the shovel invented by another firm but have a cost to train employees to use it for a specific deposit of lower grade. As technology diffuses, firms only maximize current profits in their R&D investment decisions in equilibrium.

Finally, the non-renewable resource is a homogeneous good. Despite a fully competitive resource market in the long run, firms invest in extractive technology because it is grade specific. Most similar to this understanding of innovation is Desmet and Rossi-Hansberg (2014). We abstract from other possible features like uncertainty about deposits, negative externalities from resource extraction, recycling, and short-run price fluctuations.

Our model accommodates historical trends in the prices and production of major non-renewable resources, as well as world real GDP for which we present data extending back to 1792. It implies a constant resource price equal to marginal cost over the long run. Extraction firms face constant R&D costs in converting one unit of the resource into a new reserve. This is due to the offsetting interaction between technological change and geology: (i) new extraction technology exhibits decreasing returns in making deposits of lower grades extractable; (ii) the resource quantity is geologically distributed such that it increases exponentially as the grade of its deposits decreases.

The resource price depends negatively on the average crustal concentration of the resource. For example, our model predicts that iron ore prices are on average lower than copper prices, because iron is more abundant (5 percent of crustal mass) than copper (0.007 percent). The price is also negatively affected by the average effect of technology in terms of making lower grade deposits extractable. For example, the average effect might be larger for deposits that can be extracted in open pit mines (e.g. coal) than for deposits requiring underground operations (e.g. crude oil). This implies that coal prices are lower than crude oil price in the long term.

The resource intensity of the economy, defined as the resource quantity used to produce one unit of aggregate output, is positively affected by the average geological abundance and the average effect of extraction technology, while the elasticity of substitution has a strong negative effect. If the resource and the intermediate good are complements, the resource intensity of the economy is relatively high, while it is significantly lower in the case of the two being substitutes. As the resource intensity is constant in equilibrium, firms extract the non-renewable resource at the same rate as aggregate output.

Aggregate output growth is constant on the balanced growth path. Our model predicts that a higher abundance of a particular resource or a higher average effect of extractive technology in terms of lower grades positively impact aggregate growth in the long run.

The extractive sector features only constant returns to scale. In contrast to the intermediate good sector, where firms can make use of the entire *stock* of technology for production, firms in the extractive sector can only use the *flow* of new technology to convert deposits of lower grades into new reserves. Earlier developed technologies are grade specific and the related deposits are exhausted. The stock of extraction technology therefore grows proportionally to output, while technology in the intermediate good sectors increases at the same rate as aggregate output.

The paper contributes to a literature that mostly builds on the seminal Hotelling (1931) optimal depletion model. Heal (1976) introduces a non-renewable resource, which is inexhaustible, but extractable at different grades and costs. Extraction costs increase with cumulative extraction, but then remain constant when a "backstop technology" (Heal, 1976, p. 371) is reached. Slade (1982) adds exogenous technological change in extraction technology to the Hotelling (1931) model and predicts a U-shaped relative price curve. Cynthia-Lin and Wagner (2007) use a similar model with an inexhaustible non-renewable resource and exogenous technological change. They obtain a constant relative price with increasing extraction.

There are three papers, to our knowledge, that like ours include technological change in the extraction of a non-renewable resource in an endogenous growth model. Fourgeaud et al. (1982) focuses on explaining sudden fluctuations in the development of non-renewable resource prices by allowing the resource stock to grow in a stepwise manner through technological change. Tahvonen and Salo (2001) model the transition from a non-renewable energy resource to a renewable energy resource. Their model follows a learning-by-doing approach as technological change is linearly related to the level of extraction and the level of productive capital. It explains decreasing prices and the increasing use of a non-renewable energy resource over a particular time period before prices increase in the long term. Hart (2016) models resource extraction and demand in a growth model with exogenous technological change. After a temporary "frontier phase" with a constant resource price and consumption rising at a rate only close to aggregate output, the economy needs to extract resources from greater depths. Subsequently, a long-run balanced growth path is reached with constant resource consumption and prices that rise in line with wages.

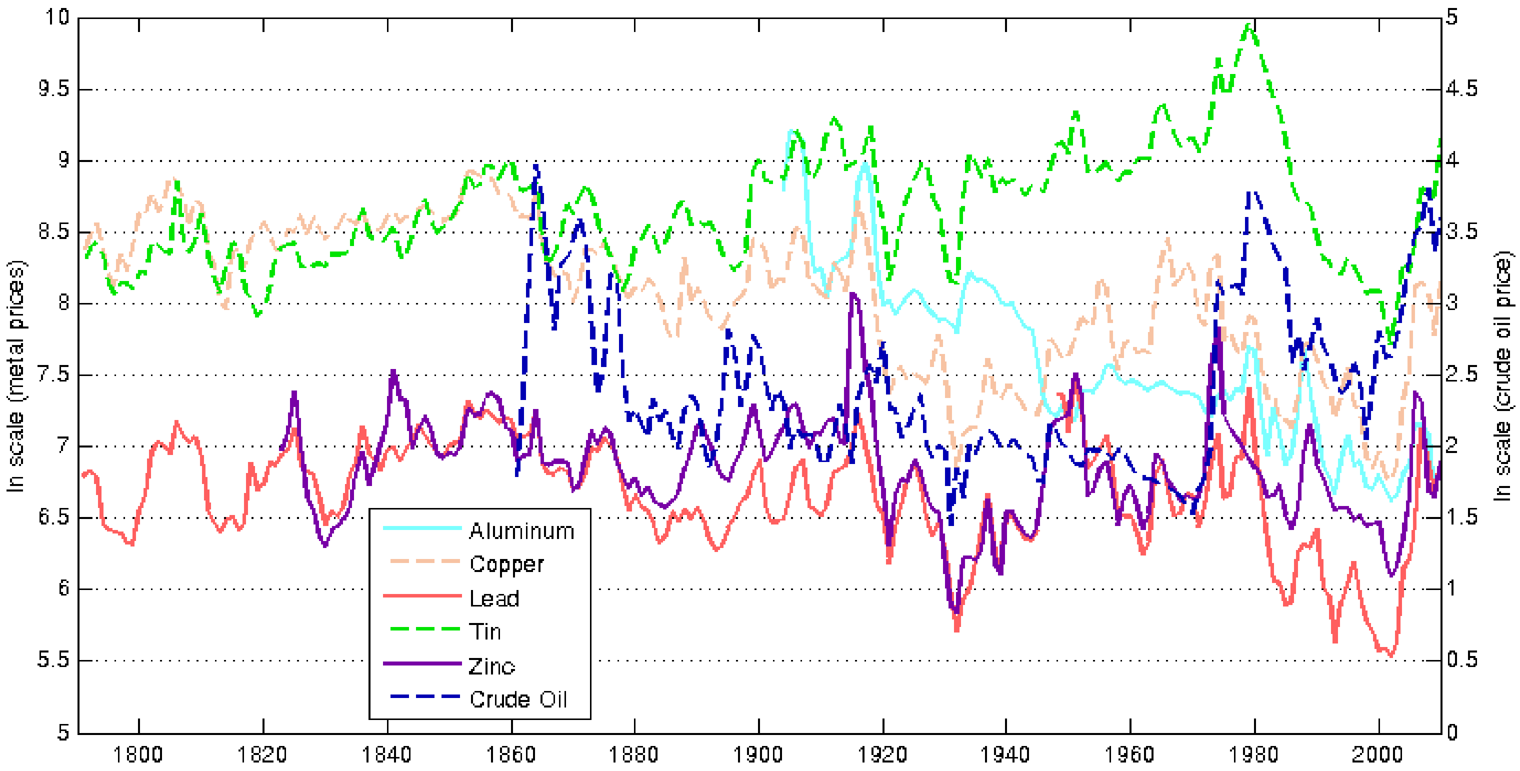
In Section 2, we document stylized facts on the long-term development of non-renewable resource prices, production, and world real GDP. We also provide evidence for the major assumptions of our model regarding geology and technological change. Section 3 introduces the main mechanisms of the theory with regard to technological change and geology. Section 4 describes the microeconomic foundations of the extractive sector and its innovation process. Section 5 presents the growth model, and section 6 derives and discusses theoretical predictions. In Section 8 we draw conclusions.

# 2 Prices, Resource Production, and Aggregate Output over the Long Term

We collect annual data for major non-renewable resource markets going back to 1792. Statistical tests indicate that real non-renewable resource prices are roughly trend-less and that worldwide primary production as well as world real GDP grow roughly at a constant rate.

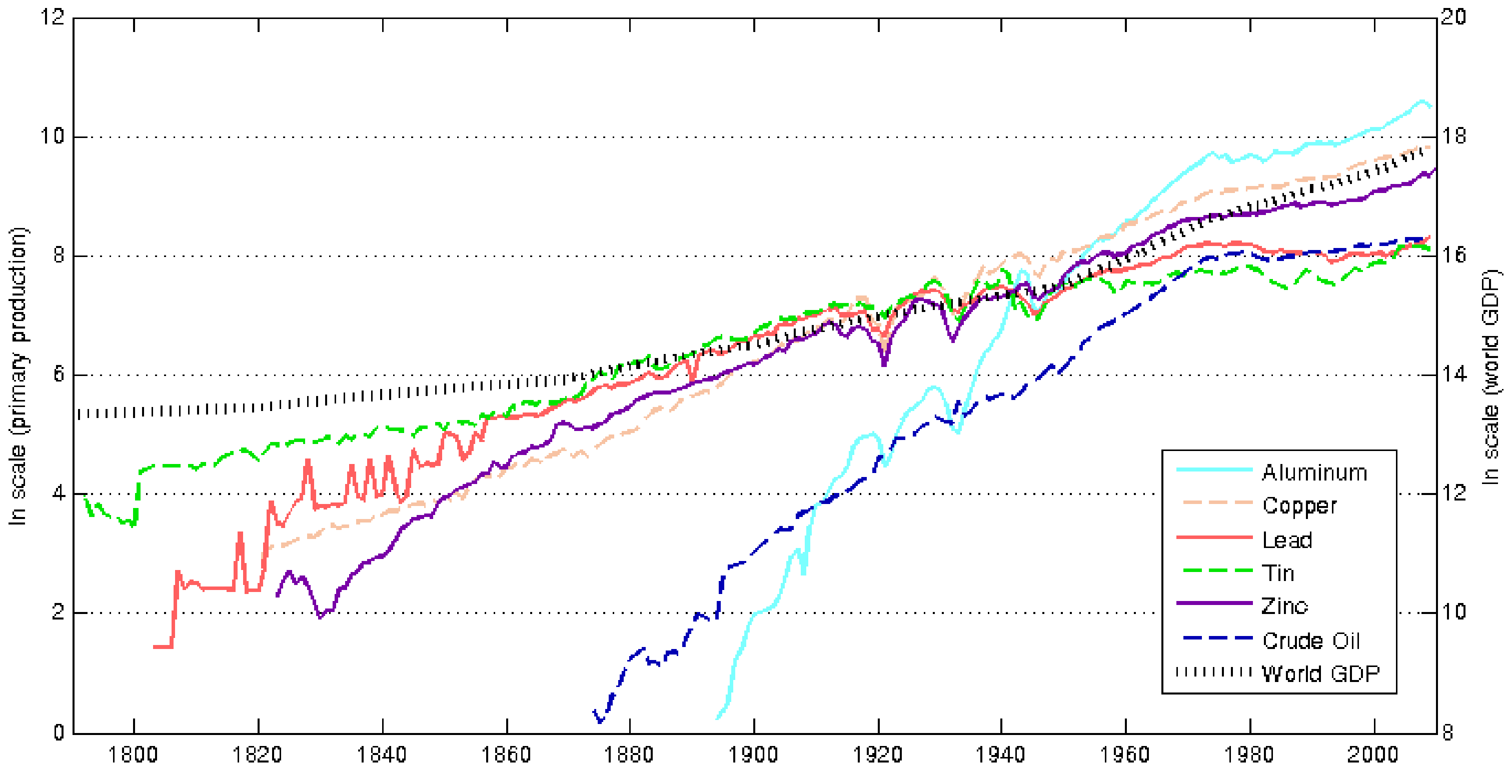
Figure 1 presents data on the real prices of five major base metals and crude oil. Real prices exhibit strong short-term fluctuations. We test the null hypothesis that the growth rates of the real prices are not significantly different from zero. As the regression results in Table 2 in the appendix show, this null hypothesis cannot be rejected. The real prices are trend-less. This is in line with evidence over other time periods provided by Krautkraemer (1998), Von Hagen (1989), Cynthia-Lin and Wagner (2007), Stuermer (2016). The real price for crude oil exhibits structural breaks over the long term, as shown in Dvir and Rogoff (2010). Overall, the literature is certainly not conclusive (see Pindyck, 1999; Lee et al., 2006; Slade, 1982; Jacks, 2013; Harvey et al., 2010), but we believe the evidence is sufficient to take trend-less prices as a motivation for our model.

Figure 2 shows that the world primary production of the examined non-renewable resources and world real GDP approximately exhibit constant positive growth rates since 1792. A closer statistical examination confirms that the production of non-renewable resources exhibits significantly positive growth rates in the long term (see table 3 in the appendix).




Notes: All prices, except for the price of crude oil, are prices of the London Metal Exchange and its predecessors. As the price of the London Metal Exchange used to be denominated in Sterling in earlier times, we have converted these prices to U.S.-Dollar by using historical exchange rates from Officer (2011). We use the U.S.-Consumer Price Index provided by Officer and Williamson (2011) and the U.S. Bureau of Labor Statistics (2010) for deflating prices with the base year 1980-82. The secondary y-axis relates to the price of crude oil. For data sources and description see Stuermer (2013).

Figure 1: Real prices of major mineral commodities in natural logs.




For data sources and description see Stuermer (2013).

Figure 2: World primary production of non-renewable resources and world real GDP in logs.

Crude oil production follows this pattern up to 1975. Inclusion of the time period from 1975 until 2009 reveals a statistically significant negative trend and, therefore, declining growth rates over time due to a structural break in the oil market (Dvir and Rogoff, 2010; Hamilton, 2009). In the case of primary aluminum production, we also find declining growth rates over time and hence, no exponential growth of the production level. This might be attributable to the increasing importance of recycling (see data by U.S. Geological Survey, 2011a).

Overall, we take these stylized facts as motivation to build a model that exhibits trend-less resource prices and constant growth in the worldwide production of non-renewable resources and in world aggregate output.

# 3 Non-Renewable Resources and Extraction Technology

Technological change in the extractive sector is different from other sectors due to its interaction with geology. As higher grade geological deposits get depleted under existing technology, firms develop new technology to convert lower grade deposits to become extractable and to ultimately continue resource production. We call this "Factor Extracting Technological Change". As the resource is a function of improving extraction technology and geology, it is like working capital. This is in contrast to factor augmenting technological change, which makes the use of a fixed factor more efficient.

In the following we introduce key concepts of our theory by describing stylized facts on the geological environment and technological change in the extractive sector and how we model them. We then lay out their interaction and introduce the concept of a non-renewable but inexhaustible resource.

## 3.1 Geological Environment

The earth's crust contains deposits of non-renewable resources, such as copper or crude oil. Table 1 shows that the crustal abundance of several major non-renewable resources is large, orders of magnitude greater than existing reserves (see also Nordhaus (1974); Aguilera et al. (2012); Rogner (1997)).

Reserves are defined as the fraction of the total resource quantity in the Earth's crust that can be economically extracted with current technology (see U.S. Geological Survey (2011c)).

| | **Reserves/ Annual production** (Years) | **Crustal abundance/ Annual production** (Years) |
|---|---|---|
| Aluminum | 139[1a] | 48,800,000,000[bc] |
| Copper | 43[a] | 95,000,000[ab] |
| Iron | 78[a] | 1,350,000,000[ab] |
| Lead | 21[a] | 70.000.000[ab] |
| Tin | 17[a] | 144.000[ab] |
| Zinc | 21[a] | 187.500.000[ab] |
| Gold | 20[d] | 27,160,000[ef] |
| Coal[2] | 129[g] | } 1,400,000[6i] |
| Crude oil[3] | 55[g] | |
| Natural Gas[4] | 59[g] | |

Notes: Definition of Reserves: "Identified resources that meet specified minimum physical and chemical criteria related to current mining and production practices and that can be economically extracted or produced at the time of determination."(Source: Schulz et al., 2017) Definition of Crustal Abundance: Total quantity of a resource in the earth crust. [1]data for bauxite, [2]includes lignite and hard coal, [3]includes conventional and unconventional oil, [4]includes conventional and unconventional gas, [5]all organic carbon in the earth's crust. Sources: [a]U.S. Geological Survey (2012b), [b]Perman et al. (2003), [c]U.S. Geological Survey (2011c),[d]U.S. Geological Survey (2011b),[e]Nordhaus (1974),[f]U.S. Geological Survey (2010), [g]Federal Institute for Geosciences and Natural Resources (2011) [g][i]Littke and Welte (1992).

Table 1: Availability of selected non-renewable resources in years of production left in the reserve and crustal mass based on current annual mine production.[1]

Non-renewable resources are not uniformly concentrated in the earth's crust. Rather, some deposits are highly concentrated with a specific resource, and other deposits are less so. In our model, we define the grade $O$ of a deposit as the average concentration of the resource; the grade ranges from 0 to 100 percent. The grade distinguishes the difficulty of extraction, where a low grade is very difficult. There are also other characteristics of mineral deposits like depth and thickness. We focus on the grade, as this is the most important characteristic.

## 3.2 Extraction Technology

Technology development in the extractive sector is special, because it is making lower grade deposits economically extractable that, due to high costs, have not been previously extractable. Technological change increases reserves (see Simpson, 1999; Nordhaus, 1974, and others). This implies that technology is *grades-specific*. Firms need to adjust their technology or make new inventions in order to extract resources from deposits of lower grades.

Empirical evidence suggests that the marginal effect of extraction technology on grades declines (see Lasserre and Ouellette, 1991; Mudd, 2007; Simpson, 1999; Wellmer, 2008). For example, Radetzki (2009) and Bartos (2002) describe how technological changes in mining equipment, prospecting, and metallurgy have gradually made possible the extraction of copper from lower grade deposits. The average ore grades of copper mines, for example, have decreased from about twenty percent 5,000 years ago to currently below one percent (Radetzki, 2009). Figure 3 illustrates this development using the example of U.S. copper mines. Gerst (2008) and Mudd (2007) come to similar results for worldwide copper mines and the mining of different base-metals in Australia.

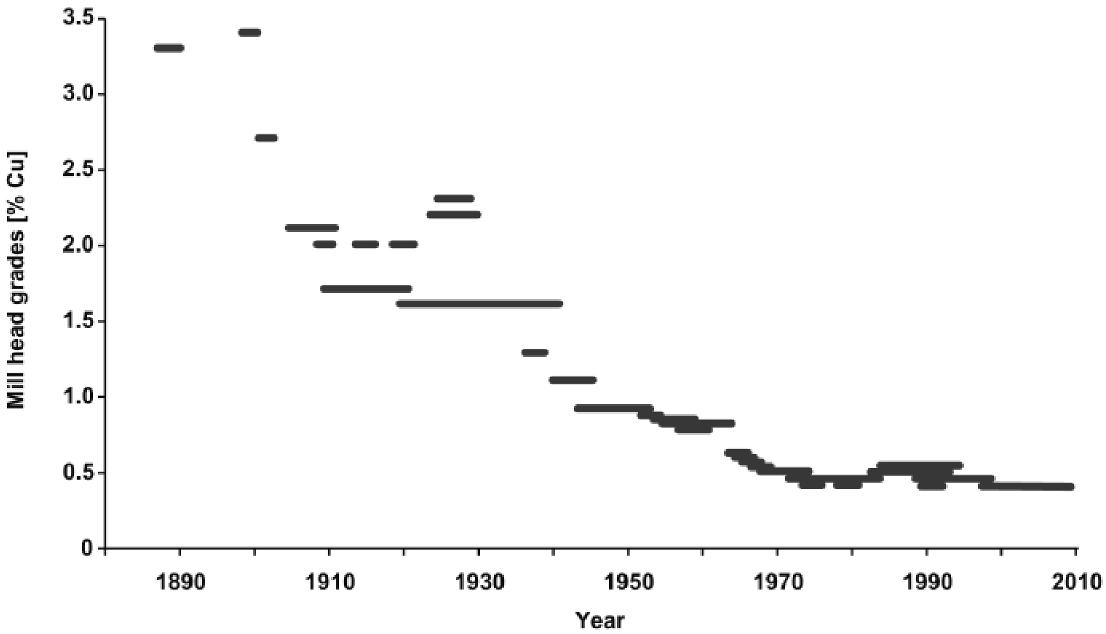



Figure 3: The historical development of mining of various grades of copper in the U.S. Source: Scholz and Wellmer (2012)

We observe similar developments for hydrocarbons. Using the example of the offshore oil industry, Managi et al. (2004) show that technological change has offset the cost-increasing degradation of resources. Crude oil has been extracted from ever deeper sources in the Gulf of Mexico, as Figure 10 in the appendix shows. Furthermore, technological change and high prices have made it profitable to extract hydrocarbons from unconventional sources, such as tight oil, oil sands, and liquid natural gas (International Energy Agency, 2012).

Figures 3 (and 10 in the appendix) also show that decreases in grades have slowed as technological development progressed. Under the reasonable assumption that global R&D investment has stayed constant or increased in real terms, there are decreasing returns to R&D in terms of making mining from deposits of lower grades economically feasible.

The extraction technology function maps the state of the extraction technology $N$ onto the extractable grade $O^\star$ of the deposits (see figure 4). The extractable grade is a decreasing convex function of technology. Technological development makes deposits economically extractable, but there are decreasing returns in terms of grades:

$$O^\star(N_R) = e^{-\mu N_R},\ \mu \in \mathbb{R}_+ \ \ N_R \in (0, \infty)\ . \tag{1}$$

The grade $O^\star$ is the lowest grade that firms can extract with technology level $N_R$. Technological change, $N_R$, expands the range of grades that can be extracted. Hence, as technology develops, the extractable ore grade falls. The curve in Figure 4 starts with deposits of close to a 100 percent ore grade, which represents the state of the world several thousand years ago. We assume that extractable ore grades only get closer to zero in the long term.

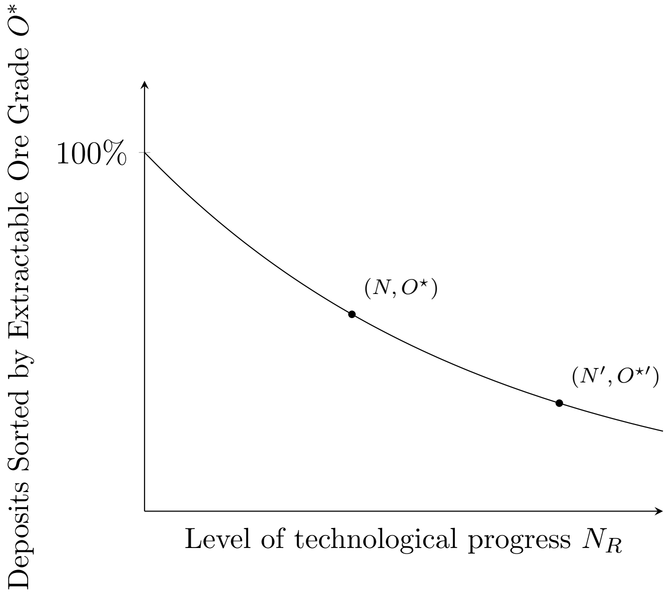



Figure 4: Extraction Technology Function.

The curvature parameter of the extraction technology function is $\mu$. If, for example, $\mu$ is high, the average effect of new technology on converting deposits to reserves in terms of grades is relatively high.

## 3.3 Geological Function

Resources are not evenly distributed across deposits in the earth's crust. Ahrens (1953, 1954) states in the fundamental law of geochemistry that each resource exhibits a log-normal grade-quantity distribution in the earth's crust, postulating a decided positive skewness.[2] Hence, the resource content of deposits increases as its grades decrease. The

[2]Geologists do not fully agree on a log-normal distribution, especially regarding very low concentrations of metals, which might be mined in the distant future. Skinner (1979) and Gordon et al. (2007) propose a discontinuity in the distribution due to the so-called "mineralogical barrier," the approximate point below which metal atoms are trapped by atomic substitution. Gerst (2008) concludes in his geological study of copper deposits that he can neither confirm nor refute these two hypotheses. However, based on worldwide data on copper deposits over the past 200 years, he finds evidence for a log-normal relationship between copper production and ore grades. Mudd (2007) analyzes the historical evolution of extraction and grades of deposits for different base metals in Australia. He finds that production has increased at a constant rate, while grades have consistently declined. We

reason is that as grades decrease deposits become larger. See figure 5 for geological evidence on copper.

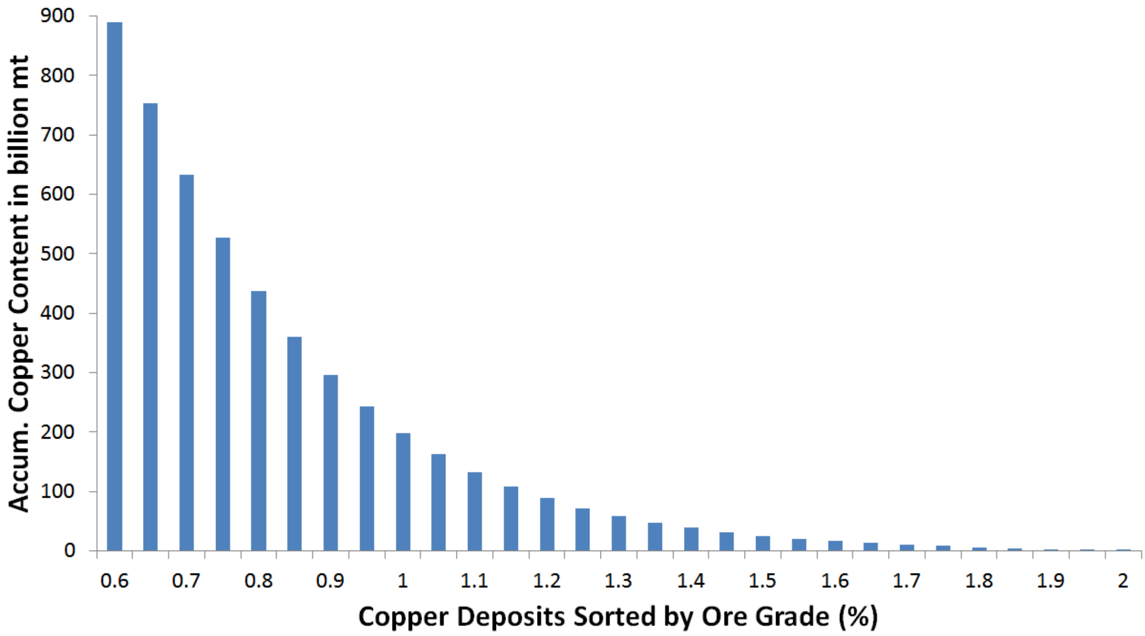



Figure 5: Cumulative grade-quantity distribution of copper in the Earth's crust. Source: Gerst (2008).

We define $Q(O^{\star})$ as the "cumulative resource quantity", that is the quantity of the resource that is (or has been - as it might have been extracted already) available in deposits of grades in the interval $[O^{\star}, 1)$. The lower bound is the lowest grade $O^{\star}$ that firms can extract with technology level $N_R$. These resources are either part of firms' reserves or have been used in past production. The geological function takes the form:

$$Q(O^{\star}) = -\delta \ln(O^{\star}), \quad \delta \in \mathbb{R}_{+} \;\; O^{\star} \in (0, 1) \; . \tag{2}$$

Figure 6 plots the function. The figure is read in direction of the red arrow. Tech-

recognize that there remains uncertainty about the geological distribution, specially regarding hydrocarbons with their distinct formation processes. However, we believe that it is reasonable to assume that a non-renewable resource is distributed according to a log-normal relationship between the grade of deposits and quantity.

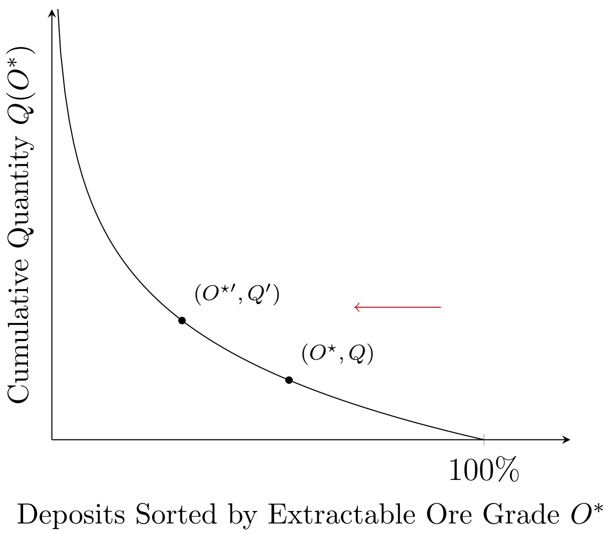



Figure 6: Geological Function.

nology development shifts the extractable deposits from grade $O^{\star}$ down to grade $O^{\star\prime}$. The respective cumulative resource quantity increases from $Q$ to $Q'$.

The functional form implies that the cumulative quantity of the resource approaches infinity as the grade of deposits gets closer to zero. Although we recognize that non-renewable resources are ultimately finite in supply, we assume that the underlying resource quantity goes to infinity for any time frame that is relevant for human economic activity. This assumption is analogue to households maximizing over an infinite horizon.

Parameter $\delta$ controls the curvature of the function. If $\delta$ is high, the marginal effect on the quantity of the non-resources from shifting to deposits of lower grades is high. It implies that the average concentration of the non-renewable resource is high in the crustal mass.

## 3.4 A Non-Renewable but Inexhaustible Resource

Technological change in resource extraction offsets the depletion of economically extractable reserves of non-renewable resources (Simpson, 1999, and others). Hence, reserves are drawn down by extraction, but increase by technological change in extraction technology.

The extraction of non-renewable resources from lower grade deposits goes hand in hand with increases of reserves over time. Figure 7 shows that copper reserves have increased by more than 700 percent since 1950. Crude oil reserves have doubled since the 1980s (see figure 11 in the appendix).[3]

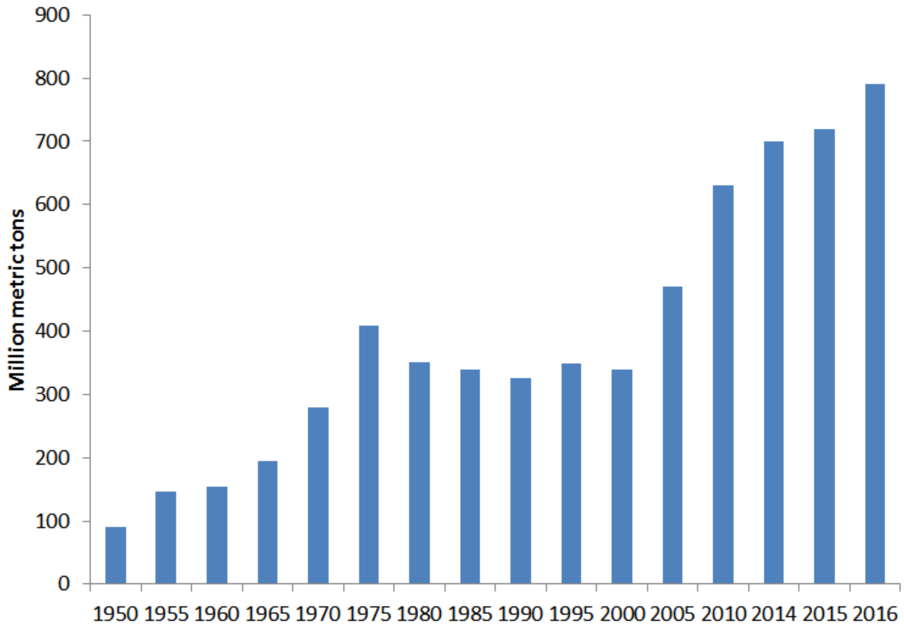



Figure 7: Historical evolution of world copper reserves from 1950 to 2016. Sources: Tilton and Lagos C.C. (2007), USGS.

[3]Note that world copper production increased by a roughly equivalent percentage since 1950, while world oil production increased by roughly 30 percent since 1980.

We propose to call a non-renewable resource, which features reserves that are drawn down by extraction, but increase by technological progress in extraction technology, a new renewable but inexhaustible resource. The traditional way of modeling non-renewable resource follows Hotelling (1931). Resource extraction $\dot{R}$ equals the draw-down of the resource stock $\dot{S}$:

$$\dot{R} = -\dot{S} \text{ with } S_t \geq 0, \dot{R}_t \geq 0 \text{ and } S_0 > 0.$$

Major assumptions of this approach are a fixed know resource stock and no extraction cost or innovation.

In contrast, extraction of the non-renewable but inexhaustible resource $\dot{R}_t$ equals the change in the reserves $\dot{S}$ and new reserves due to technological development $\dot{Q}_t$. Reserves $S$ are defined as non-renewable resource in the ground that can be extracted with current technology.

$$\dot{R}_t = -\dot{S}_t + \dot{Q}_t, \quad S_t \geq 0, \dot{Q}_t \geq 0, \dot{R}_t \geq 0. \tag{3}$$

New reserves due to technological change $\dot{Q}_t$ are a function of the extractable ore grade $O^\star$, which is a function of *grades-specific* extraction technology $N_R$:

$$\dot{Q}_t = F(O^\star(N_R)). \tag{4}$$

Note that in our model $R$ is the stock of resources that has ever been extracted. In the case of metals, these resources might either still be in use in the so called "technosphere" (partly due to recycling), in inventories, or have ended up on landfills. In the case of fossil fuels, $R$ is the stock of resources that are either still in inventories, in transportation to a combustion unit, or have been burnt. $S$ is the stock of resources in firms' reserves. $Q$ is the stock of resources that have ever been converted to reserves. Hence, $R + S = Q$. We call $Q$ the cumulative resource quantity.

## 3.5 Marginal Effect of Extraction Technology on Reserves

The technology function, equation (1), and the geology function, equation (2), have offsetting effects. This leads to a constant marginal effect of new technology on new reserves.

**Proposition 1** *The cumulative resource quantity develops proportionally to the level of extraction technology $N_R$:*

$$Q(O^{\star}(N_{Rt})) = \delta\mu N_{Rt} \ .$$

*The marginal effect of new extraction technology on the cumulative resource quantity $Q_t$ equals:*

$$\frac{dQ(O^{\star}(N_{Rt}))}{dN_R} = \delta\mu \ .$$

As the natural exponential in (1) and the natural log in equation (2) cancel out, the relationship between investment in technology and the cumulative resource quantity is linear.

**Proof of Proposition 1**

$$\begin{aligned} Q(O^\star(N_{Rt})) &= -\delta \ln(O^\star(N_{Rt})) \\ &= -\delta \ln(e^{-\mu N_{Rt}}) \\ &= \mu\delta N_{Rt} \end{aligned}$$

□

The intuition is that two offsetting effects cause this result: (i) the cumulative resource quantity is geologically distributed such that it implies increasing returns in terms of new reserves as the grade of deposits decline; (ii) new extraction technology exhibits decreasing returns in terms of making lower grade deposits extractable.

Figure 8 illustrates how the interaction of the geological and the extraction technology functions leads to a linear relationship between technology and reserves. The upper left panel shows how two equal steps in advancing technology from 0 to $N$ and from $N$ to $N'$, lead to diminishing returns in terms of extractable ore grades $O^\star$ and $O^{\star\prime}$, where $O^{\star\prime} - O^\star \prec O^\star$. The lower right panel depicts how the two related extractable ore grades $O^\star$ and $O^{\star\prime}$ map into equally sized steps in the cumulative resource quantity $Q$ and $Q'$, where $Q' - Q = Q$. Finally, the lower left panel summarizes the linear relationship between the level technological progress and the cumulative resource quantity as a result of the two functions.

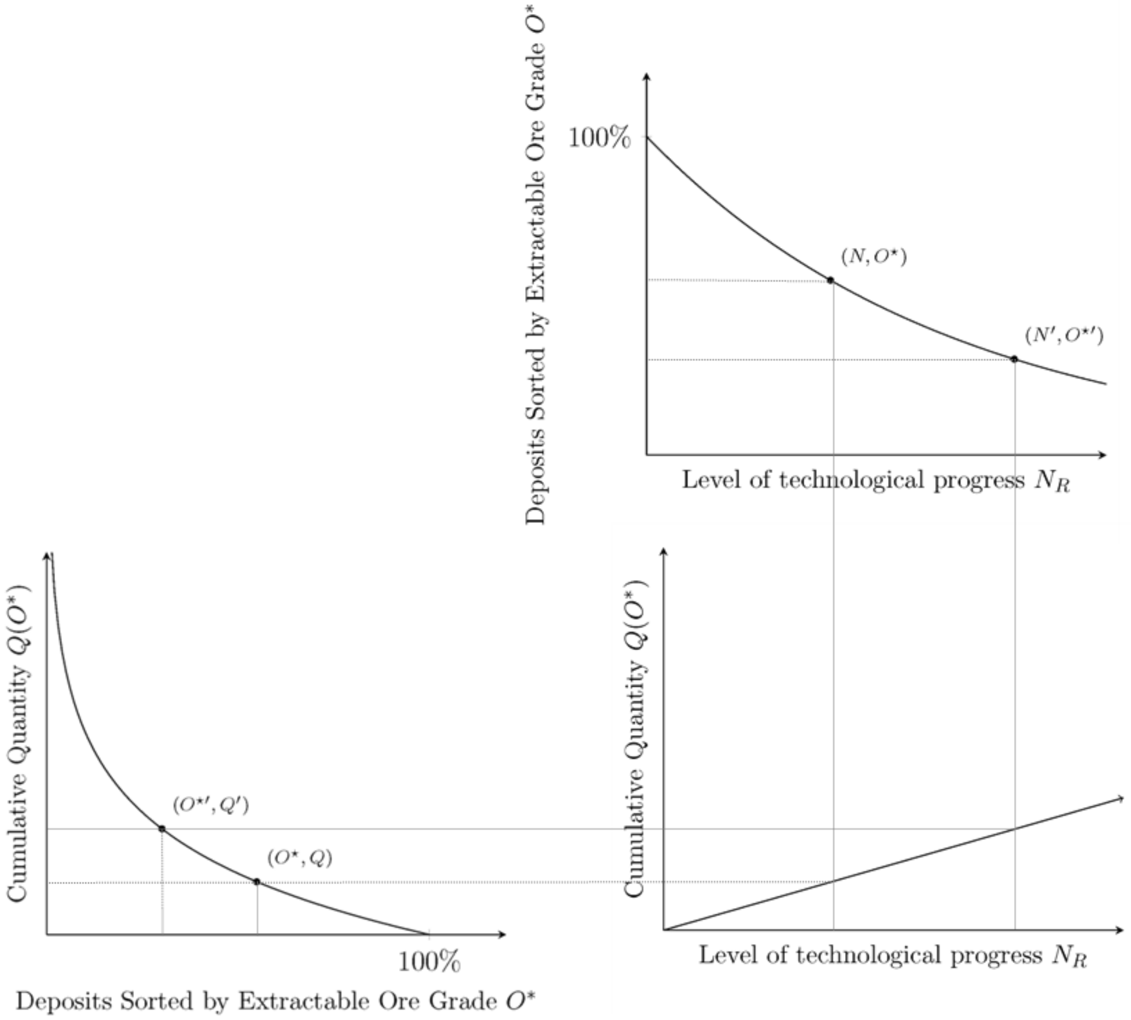



Figure 8: The interaction between the extraction technology function (upper left panel) and the extraction technology function (lower right panel) leads to a linear relationship between technology $N_R$ and cumulative resource quantity $Q$ (lower right panel).

The equations in Proposition 1 depend on the shapes of the geological function and the technology function. If the respective parameters $\delta$ and $\mu$ are high, the marginal return on new extraction technology will also be high.

The constant effect of technology on new reserves implies that the social value of an innovation is equal to the private value. R&D development does not cause an exhaustion of the resource. Future innovations are not reduced in profitability. No positive or negative spill-overs occur in our model.

# 4 The Extractive Sector

We first set up a simple extractive sector. There are two different types of firms, extraction firms and technology firms. The former buy technology from the technology firms and extract the resource, while the latter innovate and produce technology. The sector is constructed in analogy to Acemoglu (2002) to ease comparison. We use continuous time to facilitate interpretation of the necessary conditions and the analysis of equilibrium dynamics.

## 4.1 Extractive Firms

We consider a large number of infinitely small extractive firms.[4] As we model long-run trends in the extractive sector, we assume that the sector is fully competitive and firms take the demand for the non-renewable resource as given.[5] Firms fully know about the

[4]We assume that the firm level production functions exhibit constant returns to scale, so there is no loss of generality in focusing on aggregate production functions.

[5]Historically, producer efforts to raise prices were successful in some non-oil commodity markets, though short-lived as longer-run price elasticities proved to be high (see Radetzki, 2008; Herfindahl, 1959; Rausser and Stuermer, 2016). Similarly, a number of academic studies discard OPEC's ability to

distribution of the resource across deposits.

Firms use new technology to extract the resource from their reserves $S$. Reserves are defined as non-renewable resource in underground deposits that can be extracted with the *grades-specific* technology at a constant extraction cost $\phi > 0$. We assume that the marginal extraction cost for deposits not classified as reserves are infinitely high, $\phi = \infty$. Technology depreciates fully after use.

Firms can expand their reserves by investing in new *grades-specific* technology of variety $j$. Each new technology $j$ makes deposits of lower grades $O$ extractable. We assume decreasing returns of technological change in terms of ore grades (see equation (1)). Extraction firms can purchase the new technology from sector-specific technology firms at price $\chi_R$. This allows firms to claim ownership of all of the non-renewable resource in the respective additional deposits. Firms declare these deposits their new reserves.

Combining equations (3) and Proposition 1, the net rate of change of firms' reserves is:

$$\dot{S}_t = -\dot{R}_t + \dot{Q}_t \qquad S_t \geq 0, \dot{Q}_t \geq 0, \dot{R}_t \geq 0,$$

where new reserves equal:[6]

$$\dot{Q}_t = \delta\mu\dot{N}_R. \tag{5}$$

Extractive firms' profit function is: $\pi_R^E = p_R\dot{R} - \phi\dot{R} - \chi_R\delta\mu\dot{N}$,

raise prices over the long term (see Aguilera and Radetzki, 2016, for an overview). This is in line with historical evidence that OPEC has never constrained members' capacity expansions, which would be a precondition for long-lasting price interventions (Aguilera and Radetzki, 2016)

[6]Please see Appendix 1.1 for the derivation of this equation.

## 4.2 Technology Firms in the Extractive Sector

New extraction technologies are supplied by sector-specific technology firms. The innovation possibilities frontier, which determines how new technologies are created, is assumed to take the following form:[7]

$$\dot{N}_R = \eta_R M_R \; . \tag{6}$$

Technology firms can spend one unit of the final good for R&D investment $M$ at time $t$ to generate a flow rate $\eta_R > 0$ of new patents, respectively. Each technology firm can hence freely enter the market if it develops a patent for a new extraction technology (or machine) $j$ at this cost.[8] Firms enter the market until the value of entering, namely profits, equals market entry cost. The free entry condition is thus

$$\frac{1}{\eta_R} = \pi_{Rt} \; .$$

The new technology is non-rival, but excludable because it applies only to specific deposits. Technology diffuses immediately. Once a firm has invented a technology, each technology can be produced at a fixed marginal cost $\psi_R > 0$. Each technology is only produced once, as the respective deposits are depleted and new technologies need to be invented.

Even though technology firms have a monopoly on patent $j$, different machines can

[7] We assume in line with Acemoglu (2002) that there is no aggregate uncertainty in the innovation process. There is idiosyncratic uncertainty, but with many different technology firms undertaking research, equation 6 holds deterministically at the aggregate level.

[8] We use $j$ to denominate both, new machines and technology firms, because each firm can only invent one new machine in line with Acemoglu (2002).

be regarded as substitutes since they all give access to additional deposits of the same homogenous resource. As a result, technology firms in the extractive sector do not have market power. Machine prices $\chi_R(j)$ result from the market equilibrium of demand and marginal cost. Patents have still value, because extraction firms have to buy machines of varieties $j$ to extent their reserves and ultimately to continue producing.

In the extractive sector, the value of a technology firm that discovers a new machine depends hence only on instantaneous profit.

$$V_R(j) = \pi_R(j) = \chi_R(j) - \psi_R \ , \tag{7}$$

This allows us to boil down a dynamic optimization problem to a static one. It makes the model solvable and computable. At the same time, the model is rich enough to derive meaningful theoretical predictions about the relationship between technological change, geology and economic growth.

Figure 9: Timing and Firms' Problem

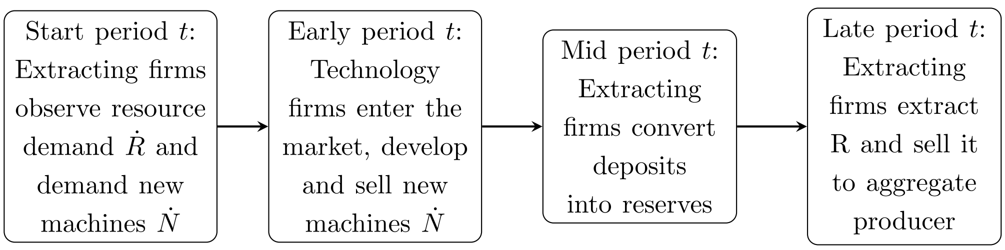



Figure 9 illustrates the timing in our model. At the start of period $t$, extraction firms observe the resource demand from the aggregate production sector and they demand new technologies from the technology firms. In the early period of $t$, technology firms observe this demand and decide if they want to invest into developing new machines and enter the market. Each firm produces one machine based on the patent that it develops and sells it to the extraction firms. In the later period of $t$, extraction firms convert deposits to reserves based on the new machines and extract the resource.

# 5 Extraction Technology in an Endogenous Growth Model

We embed the extractive sector in an endogenous growth model with two sectors, and take the framework by Romer (1986) and Acemoglu (2002) as a starting point. The general equilibrium model setup and the intermediate goods sector will be presented in this section.

## 5.1 Setup

We consider a standard setup of an economy with a representative consumer that has constant relative risk aversion preferences:

$$\int_0^\infty \frac{C_t^{1-\theta}-1}{1-\theta} e^{-\rho t} dt \; .$$

The variable $C_t$ denotes consumption of aggregate output at time $t$, $\rho$ is the discount rate, and $\theta$ is the coefficient of relative risk aversion.

The aggregate production function combines two inputs, namely an intermediate good $Z$ and a non-renewable resource $R$, with a constant elasticity of substitution:

$$
Y = \left[\gamma Z^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\dot{R}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \ . \tag{8}
$$

The distribution parameter $\gamma \in (0,1)$ indicates their respective importance in producing aggregate output $Y$. The elasticity of substitution is $\varepsilon > 0$, when the resource is not essential for aggregate production (see Dasgupta and Heal, 1980).

The budget constraint of the representative consumer is: $C+I+M \leq Y$. Aggregate spending on machines is denoted by $I$ and aggregate R&D investment by $M$, where $M = M_Z + M_R$. The usual no-Ponzi game conditions apply.

Setting the price of the final good as the numeraire gives:

$$
\left[\gamma^{\varepsilon} p_Z^{1-\varepsilon} + (1-\gamma)^{\varepsilon} p_R^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} = 1 \ , \tag{9}
$$

where $p_Z$ is the price index of the intermediate good and $p_R$ is the price index of the non-renewable resource. Intertemporal prices of the intermediate good are given by the interest rate $[r_t]_{T=0}^{\infty}$.

## 5.2 Intermediate Good Sector

The intermediate good sector follows the basic setup of Acemoglu (2002). It consists of a large number of infinitely small firms that produce the intermediate good, and technology firms that produce sector-specific technologies.[9]

[9]Like in the extractive sector, we assume that the firm level production functions exhibit constant returns to scale, so there is no loss of generality in focusing on aggregate production functions. Firms in the extractive and in the intermediate sectors use different types of machines to produce the non-renewable resource and the intermediate good, respectively. Firms are owned by the representative

Firms produce an intermediate good $Z$ according to the production function:

$$Z = \frac{1}{1-\beta_Z} \left( \int_0^{N_z} x_z(j)^{1-\beta_Z} dj \right) L_Z^{\beta} , \tag{10}$$

where $x_Z(j)$ refers to the number of machines used for each machine variety $j$ in the production of the intermediate good, $L$ is labor, which is in fixed supply, and $\beta_Z$ is $\in (0,1)$. This implies that machines in the intermediate good sector are partial complements.[10]

All intermediate good machines are supplied by sector-specific technology firms that each have one fully enforced perpetual patent on the respective machine variety. As machines are partial complements, technology firms have some degree of market power and can set the price for machines. The price charged by these firms at time $t$ is denoted $\chi_Z(j)$ for $j \in [0, N_Z(t)]$. Once invented, machines can be produced at a fixed marginal cost $\psi_Z > 0$.

The innovation possibilities frontier is assumed to take a similar form like in the extractive sector: $\dot{N}_Z = \eta_R M_Z$. Technology firms can spend one unit of the final good for R&D investment $M_Z$ at time $t$ to generate flow rate $\eta_Z > 0$ of new patents. Each firm hence needs $\frac{1}{\eta_Z}$ units of final output to develop a new machine variety. Technology firms can freely enter the market if they develop a patent for a new machine variety. They can only invent one new variety.

household.

[10]While machines of type $j$ in the intermediate sector can be used infinitely often, a machine of variety $j$ in the resource sector is grade-specific and essential to extracting the resource from deposits of certain grades $O$. A machine of variety $j$ in the extractive sector is therefore only used once, and the range of machines employed to produce resources at time $t$ is $\dot{N}_R$. In contrast, the intermediate good sector can use the full range of machines $[0, N_Z(t)]$ complementing labor.

# 6 Characterization of Equilibrium

We define the allocation in this economy by the following objects: time paths of consumption levels, aggregate spending on machines, and aggregate R&D expenditure, $[C_t, I_t, M_t]_{t=0}^{\infty}$; time paths of available machine varieties, $[N_{Rt}, N_{Zt},]_{t=0}^{\infty}$; time paths of prices and quantities of each machine, $[\chi_{Rt}(j), x_{Rt}(j)]_{j\in[0,N_{Rt}]t}^{\infty}$ and $[\chi_{Zt}(j), x_{Zt}(j)]_{j\in[0,N_{Zt}],t}^{\infty}$; the present discounted value of profits $V_R$ and $V_Z$, and time paths of interest rates and wages, $[r_t, w_t]_{t=0}^{\infty}$.

An equilibrium is an allocation in which all technology firms in the intermediate good sector choose $[\chi_{Zt}(j), x_Zt(j)]_{j\in[0,N_Z(t)],t}^{\infty}$ to maximize profits. Machine prices in the extractive sector $\chi_{Rt}(j)$ result from the market equilibrium, because extraction technology firms are in full competition and only produce one machine per patent.

The evolution of $[N_{Rt}, N_{Zt}]_{t=0}^{\infty}$ is determined by free entry; the time paths of factor prices, $[r, w]_{t=0}^{\infty}$, are consistent with market clearing; and the time paths of $[C_t, I_t, M_t]_{t=0}^{\infty}$ are consistent with household maximization.

## 6.1 The Final Good Producer

The final good producer demands the intermediate good and the resource for aggregate production. Prices and quantities for both are determined in a fully competitive equilibrium. Taking the first order condition with respect to the intermediate good and the non-renewable resource in (8), we obtain the demand for the intermediate good

$$Z = \frac{Y(1-\gamma)^{\varepsilon}}{p_Z^{\varepsilon}} \ ,$$

and the demand for the resource

$$\dot{R} = \frac{Y(1-\gamma)^{\varepsilon}}{p_R^{\varepsilon}} \ . \tag{11}$$

## 6.2 Extraction Firms

To characterize the (unique) equilibrium, we first determine the demand for machine varieties in the extractive sector.[11] Machine prices and the number of machine varieties are determined in a market equilibrium between extractive firms and technology firms. Firms optimization problem is static since machines depreciate fully after use.

In equilibrium, it is profit maximizing for firms to not keep reserves, $S(j) = 0$.[12] It follows that the production function of extractive firms is

$$\dot{R}_t = \delta\mu\dot{N}_{Rt}. \tag{12}$$

Extractive firms face a cost for producing $\dot{R}_t$ units of resource given by $\Omega(\dot{R}_t) = \dot{R}_t\chi_R\frac{1}{\delta\mu}$, where $\chi_R$ is the machine price charged by the extraction technology firms. The marginal cost is $\Omega'(\dot{R}_t) = \chi_R\frac{1}{\delta\mu}$. The inverse supply function of the resource is

[11]Please see Appendix 1.2 for the respective derivations regarding intermediate good firms.

[12]If we assumed stochastic technological change, extractive firms would keep a positive stock of reserves $S_t$ to insure against a series of bad draws in R&D. Reserves would grow over time in line with aggregate growth. The result would, however, remain the same: In the long term, resource extraction equals new reserves.

hence constant and we obtain a market equilibrium at

$$p_R = \chi_R \frac{1}{\delta\mu}$$

and

$$\dot{R}_t = \frac{Y(1-\gamma)^{\varepsilon}}{(\chi_R \frac{1}{\delta\mu})^{\varepsilon}} \ . \tag{13}$$

Using (12), we obtain the demand for machines:

$$\dot{N}_R = \frac{1}{\delta\mu} \frac{Y(1-\gamma)^{\varepsilon}}{(\chi_R \frac{1}{\delta\mu})^{\varepsilon}} \ . \tag{14}$$

## 6.3 Technology Firms in the Extractive Sector

In the extractive sector, the demand function for extraction technologies (14) is isoelastic, but there is perfect competition between the different suppliers of extraction technologies, as machine varieties are perfect substitutes.[13] Because only one machine is produced for each machine variety $j$, the constant rental rate $\chi_R$ that all monopolists $j \in [N_{t-h}, N_t] \lim_{h \to 0}$ charge includes the cost of machine production $\psi_R$ and a mark-up that refinances R&D costs. The rental rate is the result of a competitive market and derived from (13). It equals:

$$\chi_R(j) = \left(Y/\dot{R}\right)^{\frac{1}{\varepsilon}} (1-\gamma)\delta\mu. \tag{15}$$

To complete the description of equilibrium on the technology side, we impose the

[13]Please see Appendix 1.3 for the respective derivations for technology firms in the intermediate good sector.

free-entry condition, (4.2). Like in the intermediate sector, markups are used to cover technology expenditure in the extractive sector. Combining equations (7) and (15), we obtain that the net present discounted value of profits of technology firms from developing one new machine variety is:

$$V_R(j) = \pi_R(j) = \chi_R(j) - \psi_R = \left(Y/\dot{R}\right)^{\frac{1}{\varepsilon}} (1-\gamma)\delta\mu - \psi_R \ . \tag{16}$$

To compute the equilibrium quantity of machines and machine prices in the extractive sector, we first rearrange (16) with respect to $R$ and consider the free entry condition. We obtain

$$\dot{R}_t = \frac{Y(1-\gamma)^{\varepsilon}}{\left(\left(\frac{1}{\eta_R} + \psi_R\right)\frac{1}{\delta\mu}\right)^{\varepsilon}} \ . \tag{17}$$

We insert (17) into (15) and obtain the equilibrium machine price.

$$\chi_R(j) = \frac{1}{\eta_R} + \psi_R \ . \tag{18}$$

## 6.4 Equilibrium Resource Price

The resource price equals marginal production costs due to perfect competition in the resource market. Equation (18) implies the following proposition:[14]

**Proposition 2** *The resource price depends negatively on the average crustal concentration of the non-renewable resource and the average effect of extraction technology:*

$$p_R = \left(\frac{1}{\eta_R} + \psi_R\right)\frac{1}{\delta\mu} \ , \tag{19}$$

[14]Please see Appendix 1.4 for the equilibrium price of the intermediate good.

where $\psi_R$ reflects the marginal cost of producing the machine and $\eta_R$ is a markup that serves to compensate technology firms for R&D cost.

The intuition is as follows: If, for example, $\delta$ is high, the average crustal concentration of the resource is high (see equation (2)) and the price is low. If $\mu$ is high, the average effect of new extraction technology on converting deposits of lower grades to reserves is high (see equation (1)). This implies a lower resource price. The resource price level also depends negatively on the cost parameter of R&D development $\eta_R$.

## 6.5 Resource Intensity of the Economy

Substituting equation (19) into the resource demand equation (11), we obtain the ratio of resource consumption to aggregate output.

**Proposition 3** *The resource intensity of the economy is positively affected by the average crustal concentration of the resource and the average effect of extraction technology:*

$$\frac{\dot{R}}{Y} = (1-\gamma)^{\varepsilon}\left[(\frac{1}{\eta_R} + \psi_R)\frac{1}{\delta\mu}\right]^{-\varepsilon} .$$

*The resource intensity of the economy is negatively affected by the elasticity of substitution if* $(1-\gamma)^{\varepsilon}\left[(\frac{1}{\eta_R} + \psi_R)\frac{1}{\delta\mu}\right]^{-\varepsilon} < 1$ *and positively otherwise.*

## 6.6 The Growth Rate on the Balanced Growth Path

We define the BGP equilibrium as an equilibrium path where consumption grows at the constant rate $g^*$ and the relative price $p$ is constant. From (9) this definition implies that $p_{Zt}$ and $p_{Rt}$ are also constant.

**Proposition 4** *There exists a unique BGP equilibrium in which the relative technologies are given by equation* (32) *in the appendix, and consumption and output grow at the rate*[15]

$$g = \theta^{-1}\left(\beta\eta_Z L\left[\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\frac{1}{\eta_R\delta\mu} + \frac{\psi_R}{\delta\mu}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}\frac{1}{\beta}} - \rho\right). \tag{20}$$

The growth rate of the economy is positively influenced by (i) the crustal concentration of the non-renewable resource $\delta$ and (ii) the effect of R&D investment in terms of lower ore grades $\mu$.

Adding the extractive sector to the standard model by Acemoglu (2002), changes the interest part of the Euler equation, $g = \theta^{-1}(r-\rho)$.[16] Instead of two exogenous production factors, the interest rate $r$ in our model only includes labor, but adds the resource price, as $p_Z$ depends on $p_R$ according to equation (30).

If $(1-\gamma)^{\varepsilon}(\eta_R\delta\mu)^{1-\varepsilon} < 1$ holds, then the substitution between the intermediate good and the resource is low and R&D investment in extraction technology have a small yield in terms of additional reserves. The effect that economic growth is impossible if the resource cannot be substituted by other production factors is known as the "limits to growth" effect in the literature (see Dasgupta and Heal, 1979, p. 196 for example). When the effect occurs, growth is *limited* in models with a positive initial stock of

[15]Starting with any $N_R(0) > 0$ and $N_Z(0) > 0$, there exists a unique equilibrium path. If $N_R(0)/N_Z(0) < (N_R/N_Z)^*$ as given by (32), then $M_{Rt} > 0$ and $M_{Zt} = 0$ until $N_{Rt}/N_{Zt} = (N_R/N_Z)^*$. If $N_R(0)/N_Z(0) > (N_R/N_Z)^*$, then $M_{Rt} = 0$ and $M_{Zt} > 0$ until $N_{Rt}/N_{Zt} = (N_R/N_Z)^*$. It can also be verified that there are simple transitional dynamics in this economy whereby starting with technology levels $N_R(0)$ and $N_Z(0)$, there always exists a unique equilibrium path, and it involves the economy monotonically converging to the BGP equilibrium of (20) like in Acemoglu (2002).

[16]There is no capital in this model, but agents delay consumption by investing in R&D as a function of the interest rate.

resources, because the initial resource stock can only be consumed in this case. In our model, growth is *impossible*, because there is no initial stock and the economy is not productive enough to generate the necessary technology. When the inequality does not hold, the economy is on a balanced growth path.

### 6.7 Technology Growth

We derive the growth rates of technology in the two sectors from equations (12), (11), and (19). The stock of technology in the intermediate good sector grows at the same rate as the economy.

**Proposition 5** *The stock of extraction technology grows proportionally to output according to:*

$$\dot{N}_R = (1-\gamma)^{\varepsilon} Y \left(1/\eta_R + \psi_R\right)^{-\varepsilon} (\delta\mu)^{\varepsilon-1} \, .$$

In contrast to the intermediate good sector, where firms can make use of the *stock* of technology, firms in the extractive sector can only use the *flow* of new technology to convert deposits of lower grades into new reserves. Previously developed technology cannot be employed because it is grade specific, and deposits of that particular grade have already been depleted. Note also that firms in the extractive sector need to invest a larger share of total output to attain the same rate of growth in technology in comparison to firms in the intermediate good sector.

The effects of the two parameters $\delta$ from the geological function and $\mu$ from the extraction technology function on $\dot{N}_R$ depend on the elasticity of substitution $\varepsilon$. Like in Acemoglu (2002), there are two opposing effects at play: the first is a price effect.

Technology investments are directed towards the sector of the scarce good. The second is a market size effect, meaning that technology investments are directed to the larger sector.

If the goods of the two sectors are complements ($\varepsilon < 1$), the price effect dominates. An increase in $\delta$ or $\mu$ lowers the cost of resource production and the resource price, but the technology growth rate in the resource sector decelerates, because R&D investment is directed towards the complementary intermediate good sector. If the resource and the intermediate good are substitutes ($\varepsilon > 1$), the market size effect dominates. An increase in $\delta$ or $\mu$ makes resources cheaper and causes an acceleration in the technology growth rate in the resource sector, because more of the lower cost resource is demanded.

# 7 The Case of Multiple Resources

We now extend the model and replace the generic resource with a set of distinct resources. We do so in analogy to a generic capital stock as in many growth models. We define resources $\dot{R}^{Mult}$, resource prices $p_R^{Mult}$ and resource investments $M_R^{Mult}$ as aggregates of the respective variables of different resources $i \in [0, G]$,

$$
\begin{aligned}
\dot{R}^{Mult} &= \left( \sum_i \dot{R}_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \\
p_R^{Mult} &= \left( \sum_i \frac{\dot{R}_i}{\dot{R}} p_{R_i}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{1-\sigma}}, \\
M_R^{Mult} &= \sum_i M_{R_i}, \\
\frac{\dot{R}}{Y} &= (1-\gamma)^{\epsilon} p_R^{Mult-\epsilon}, \\
g &= \theta^{-1} \left( \beta \eta_Z L \left[ \gamma^{-\varepsilon} - \left( \frac{1-\gamma}{\gamma} \right)^{\varepsilon} p_R^{Mult 1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}\frac{1}{\beta}} - \rho \right),
\end{aligned}
$$

where $\sigma$ is the elasticity of substitution between the different resources. Note that the aggregate resource price consists of the average of the individual resources weighted by their share in physical production.

This extension can be used to make theoretical predictions. As an example, we focus here on the relative price of two resources, aluminum $a$ and copper $c$. Using equation (19) and assuming that the cost of producing machines $\psi_R$ and the flow rate of innovations $\eta_R$ are uniform across resources,we obtain that prices depend solely on geological and technological parameters:

$p_R^c = (\delta^c \mu^c)^{-1}$ and $p_R^a = (\delta^a \mu^a)^{-1}$.

Total resource production equals

$$
\dot{R} = \left( \dot{R}^{c\frac{\sigma-1}{\sigma}} + \dot{R}^{a\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},
$$

From this, we derive the following theoretical predictions:

$\frac{p_R^c}{p_R^a} = \frac{(\delta^a \mu^a)}{(\delta^c \mu^c)}$

and $\frac{\dot{R}^c}{\dot{R}^a} = (\frac{(\delta^c \mu^c)}{(\delta^a \mu^a)})^{\sigma}$,

$\frac{p_R^c \dot{R}^c}{p_R^a \dot{R}^a} = (\frac{\delta^a \mu^a}{\delta^c \mu^c})^{\sigma-1}$ and $\frac{\dot{N}_R^c}{\dot{N}_R^a} = (\frac{\delta^c \mu^c}{(\delta^a \mu^a)})^{\sigma-1}(\frac{\eta_R^c}{\eta_R^a})^{\sigma}$

We can investigate what happens when a new resource gets used (e.g. aluminum was not used until the end of the $XIX^{th}$). If we assume that $\sigma > 1$ and that the resource is immediately at its steady-state price, the price of the resource aggregate will immediately decline and the growth rate of the economy will increase: $p_R = ((\delta_1^c \delta_2^c)^{\sigma-1} + (\delta_1^a \delta_2^a)^{\sigma-1})^{\frac{1}{1-\sigma}}$.

Alternatively, a progressive increase in aluminum technology, $\dot{N}_R^a = \eta_R^a \min{(N_R^a/\overline{N}, 1)}$ $M_R^a$, would generate an initial decline in the real price (as $\eta_R^a \min{(N_R^a/\overline{N}, 1)}$ increases) and faster growth in the use of aluminum initially. This is in line with historical evidence from the copper and aluminum markets.

## 7.1 Discussion

We discuss the assumptions made in section 5, the comparison to other models with non-renewable resources, and the ultimate finiteness of the resource.

We chose the functional forms of the geological function and the extraction technology based on empirical evidence. Our model provides theoretical results that are consistent with the historical evolution resource prices and production. However, for making long-term predictions based on our model, a natural question is wow other functional forms of the two functions would affect the predictions of the model. First, if any of the two function is discontinuous with an unanticipated break, at which the respective parameters changes to either $\delta' \in \mathbb{R}_+$ or $\mu' \in \mathbb{R}_+$, there will be two balanced growth paths: one for the period before, and one for the period after the break. Both paths would behave according to the model's predictions. As an illustration, assume that $\delta' > \delta$. According to proposition (1), the amount of resources in reserved obtained per unit of investment into extraction technology would increase. This would lower the resource price in proposition (2), increase the resource intensity in proposition (3), and increase the growth rate of the economy (see proposition (26)).

Second, if one or both of them has a different form, the effects on resource price, resource intensity of the economy, and growth rate will depend on the resulting changes for proposition 1. Intuitively, if the increasing returns in the geology function do not offset the decreasing returns in the technology function, the resource price will increase over time, the resource intensity will decline and the growth rate of the economy will decline as well. There will still be no scarcity rent like in Hotelling (1931)[17], because firms continue to extract resources in a competitive market and firms cannot take prices above marginal cost.

If the increasing returns in the geology function more than offset the decreasing returns in the technology function, the resource price will decline and the resource becomes more abundant. As a result, the resource price will decline, the resource intensity increase, and the growth rate of the economy will go up. Our model can also be generalized to this case, since the condition that resource prices equal marginal resource extraction cost would extend to this case. Prices cannot be below marginal extraction cost, since firms would make negative profits. Different forms of the function

[17]Note that a scarcity rent has not yet been found empirically (see e.g. Hart and Spiro, 2011)

could also lead to a mixture of these two cases. For example, the resource price increases for some time and then declines. This would cause a declining and then increasing resource intensity and growth rate of the economy.

How does our model compare to other models with non-renewable resources? We make the convenient assumption that the quantity of non-renewable resources is for all practical economic purposes approaches infinite. As a consequence, resource availability does not limit growth if there is investment in technological change. Substitution of capital for non-renewable resources, technological change in the use of the resource, and increasing returns to scale are therefore not necessary for sustained growth as in Groth (2007) or Aghion and Howitt (1998). If the resource was finite in our model, the extractive sector would behave in the same way as in standard models with a sector based on Hotelling (1931). As Dasgupta and Heal (1980) point out, in this case the growth rate of the economy depends strongly on the degree of substitution between the resource and other economic inputs. For $\varepsilon > 1$, the resource is non-essential; for $\varepsilon < 1$, the total output that the economy is capable of producing is finite. The production function is, therefore, only interesting for the Cobb-Douglas case.

Our model suggests that the non-renewable resource can be thought of as a form of capital: if the extractive firms invest in R&D in extraction technology, the resource is extractable without limits as an input to aggregate production. This feature marks a distinctive difference from models such as the one of Bretschger and Smulders (2012). They investigate the effect of various assumptions about substitutability and a decentralized market on long-run growth, but keep the assumption of a finite non-renewable resource. Without this assumption, the elasticity of substitution between the non-renewable resource and other input factors is no longer central to the analysis of limits to growth.

Some might argue that the relationship described in proposition 1 cannot continue to hold in the future as the amount of non-renewable resources in the earth's crust is ultimately finite. Scarcity will become increasingly important, and the scarcity rent will be positive even in the present. However, for understanding current prices and consumption patterns, current expectations about future developments are important. Given that the quantities of available resources indicated in table 1 are very large, their ultimate end far in the future should approximately not affect economic behavior today and in the near future. The relationship described in proposition 1 seems to have held in the past and looks likely to hold for the foreseeable future. Since in the long term, extracted resources equal the resources added to reserves due to R&D in extraction technology, the price for a unit of the resource will equal the extraction cost plus the per-unit cost of R&D and hence, stay constant in the long term. This may explain why scarcity rents cannot be found empirically.

# 8 Conclusion

This paper examines interaction between geology and technology and its impact on the resource price, total output growth, and the resource intensity of the economy. We argue that economic growth causes the production and use of a non-renewable resource to increase at a constant rate. The marginal production cost of non-renewable resources stay constant in the long term. Economic growth enables firms to invest in extraction technology R&D, which makes resources from deposits of lower grades economically extractable. We help explain the long-term evolution of non-renewable resource prices and world production for more than 200 years. If historical trends in technological progress continue, it is possible that non-renewable resources are, within a time frame relevant for humanity, practically inexhaustible.

Our model makes simplifying but reasonable assumptions, which render our model analytically solvable. However, we believe that a less simple model would essentially provide the same results. There are four major simplifications in our model, which should be examined in more detail in future extensions. First, there is no uncertainty in R&D development, and therefore no incentive for firms to keep a positive amount of the non-renewable resource in their reserves. If R&D development is stochastic as in Dasgupta and Stiglitz (1981), there would be a need for firms to keep reserves.

Second, our model features perfect competition in the extractive sector. We could obtain a model with monopolistic competition in the extractive sector by introducing explicitly privately-owned deposits. A firm would need to pay a certain upfront cost or exploration cost in order to acquire a mineral deposit. This upfront cost would give technology firms a certain monopoly power as they develop machines that are specific to a single deposit.

Third, extractive firms could face a trade-off between accepting high extraction costs due to a lower technology level and investing in R&D to reduce extraction costs. A more general extraction technology function would provide the basis to generalize this assumption.

Fourth, our model does not include recycling. Recycling has become more important for metal production over time due to the increasing abundance of recyclable materials and the comparatively low energy requirements (see Wellmer and Dalheimer, 2012). Introducing recycling into our model would further strengthen the argument of this paper, as it increases the economically extractable stock of the non-renewable resource.

Finally, firms' holdings of reserves are zero in our model owing to the constant price and no uncertainty about research outcomes. We leave it to future work to lift the assumption of no aggregate uncertainty and to model positive reserve holdings, as we observe than empirically.

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# Appendix 1

## Appendix 1.1 Derivation of Extraction Firms' New Reserves

Equation (5) is derived in the following way: Firms can buy machines $j$ to increase their reserves by:

$$\dot{Q}_t = \delta\mu \lim_{h\to 0} \frac{1}{h} \int_{N_R(t-h)}^{N_R(t)} x_R(j)^{(1-\beta)} dj \ , \tag{21}$$

where $x_R(j)$ refers to the number of machines used for each machine variety $j$.

We assume that $\beta = 0$ in the extractive sector, because firms invest into technology to continue resource production. If firms do not invest, extraction cost becomes infinitely high. Firms are indifferent from which deposits they extract the resource. A machine of variety $j$ in the resource sector is grade-specific and essential to extracting the resource from deposits of certain grades. A machine of variety $j$ in the extractive sector is therefore only used once, and the range of machines employed to produce resources at time $t$ is $\dot{N}_R$. In contrast, the intermediate good sector can use machine types infinitely often and hence the full range of machines $[0, N_Z(t)]$ complementing labor. Under the assumption that $x_R(j) = 1$, equation (21) turns into:

$$\begin{aligned} \dot{Q}_t &= \delta\mu \lim_{h\to 0} \frac{1}{h} \int_{N_R(t-h)}^{N_R(t)} 1 dj \\ &= \delta\mu \dot{N}_R \ . \end{aligned}$$

## Appendix 1.2 Solving for the Equilibrium: Intermediate Good Firms

For the intermediate good firms, the maximization problem can be written as

$$\max_{L,\{x_Z(j)\}j\in[0,N_{Zt}]} p_Z Z - wL - \int_0^{N_Z} \chi_Z(j) x_Z(j) dj \ .$$

The problem is static, as machines depreciate fully.

The FOC with respect to $x_Z(j)$ immediately implies the following isoelastic demand function for machines:

$$x_{Zt}(j) = \left(\frac{p_{Zt}}{\chi_{Zt}(j)}\right)^{1/\beta} L \ , \tag{22}$$

for all $j \in [0, N_Z(t)]$ and all $t$,

## Appendix 1.3 Solving for the Equilibrium: Technology Firms in the Intermediate Good Sector

Substituting (22) into (23), we calculate the FOC with respect to machine prices in the intermediate good sector: $\chi_Z(j)$: $\left(\frac{p_Z}{\chi_Z(j)}\right)^{\frac{1}{\beta}} L - (\chi_Z(j) - \psi_R) p_Z^{\frac{1}{\beta}} \frac{1}{\beta} \chi_Z(j)^{\frac{1}{\beta}-1} L = 0$. Hence, the solution of the maximization problem of any monopolist $j \in [0, N_Z]$ involves setting the same price in every period according to

$$\chi_{Zt}(j) = \frac{\psi_R}{1-\beta} \text{ for all } j \text{ and } t \ .$$

The value of a technology firm in the intermediate good sector that discovers one of the machines is given by the standard formula for the present discounted value of profits:

$$V_Z(j) = \int_t^\infty exp\left(-\int_t^s r(s')ds'\right)\pi_Z(j)ds \ .$$

Instantaneous profits are denoted

$$\pi_Z(j) = (\chi_Z(j) - \psi_Z)x_Z(j) \ , \tag{23}$$

where $r$ is the market interest rate, and $x_Z(j)$ and $\chi_Z(j)$ are the profit-maximizing choices for the technology monopolist in the intermediate good sector.

All monopolists in the intermediate good sector charge a constant rental rate equal to a markup over their marginal cost of machine production, $\psi_R$. We normalize the marginal cost of machine production to $\psi_R \equiv (1-\beta)$ (remember that the elasticity of substitution between machines is $\epsilon \equiv \frac{1}{\beta}$), so that

$$\chi_{Zt}(j) = \chi_Z = 1 \text{ for all } j \text{ and } t \ . \tag{24}$$

In the intermediate good sector, substituting the machine prices (24) into the demand function (22) yields: $x_{Zt}(j) = p_{Zt}^{1/\beta}L$ for all $j$ and all $t$.

Since the machine quantities do not depend on the identity of the machine, only on the sector that is being served, profits are also independent of machine variety in both sectors. Firms are symmetric.

In particular profits of technology firms in the intermediate good sector are $\pi_{Zt} = \beta p_{Zt}^{1/\beta} L$. This implies that the net present discounted value of monopolists only depends on the sector and can be denoted by $V_{Zt}$.

Combining the demand for machines (22) with the production function of the intermediate good sector (10) yields the *derived* production function:

$$Z(t) = \frac{1}{1-\beta} p_{Zt}^{\frac{1-\beta}{\beta}} N_{Zt} L, \tag{25}$$

The equivalent equation in the extractive sector is (12), because there is no optimization over the number of machines by the extraction technology firms, as the demand for machines per machine variety is one.

## Appendix 1.4 Equilibrium Prices

Prices of the intermediate good and the non-renewable resource are derived from the marginal product conditions of the final good technology, (8), which imply

$$\begin{aligned} p &\equiv \frac{p_R}{p_Z} = \frac{1-\gamma}{\gamma} \left( \frac{\dot{R}}{Z} \right)^{-\frac{1}{\varepsilon}} \\ &= \frac{1-\gamma}{\gamma} \left( \frac{\delta \mu \dot{N}_R}{\frac{1}{1-\beta} p_L^{\frac{1-\beta}{\beta}} N_Z L} \right)^{-\frac{1}{\varepsilon}} \end{aligned}$$

There is no derived elasticity of substitution in analogy to Acemoglu (2002), because there is only one fixed factor, namely $L$ in the intermediate good sector. In the extractive sector, resources are produced by machines from deposits. The first line of this expression simply defines $p$ as the relative price between the intermediate good and the non-renewable resource, and uses the fact that the ratio of the marginal productivities of the two goods must be equal to this relative price. The second line substitutes from (25) and (12). There are no relative factor prices in this economy like in Acemoglu (2002), because there is only one fixed factor in the economy, namely $L$ in the intermediate good sector.

## Appendix 1.5 Proof for the Balanced Growth Path

We define the BGP equilibrium as an equilibrium path where consumption grows at the constant rate $g^*$ and the relative price $p$ is constant. From (9) this definition implies that $p_{Zt}$ and $p_{Rt}$ are also constant.

Household optimization implies

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\theta}(r_t - \rho),$$

and

$$\lim_{t\to\infty} \left[ \exp\left( -\int_0^t r(s)ds \right) (N_{Zt}V_{Zt} + \dot{N}_{Rt}V_{Rt}) \right] = 0,$$

which uses the fact that $N_{Zt}V_{Zt} + \dot{N}_{Rt}V_{Rt}$ is the total value of corporate assets in the economy. In the resource sector, only *new* machine varieties produce profit.

The consumer earns wages from working in the intermediate good sector and earns interest on investing in technology $M_Z$. The budget constraint thus is $C = wL + rM_Z$. Maximizing utility in equation (5.1) with respect to consumption and investments yields the first order conditions $C^{-\theta}e^{-\rho t} = \lambda$ and $\dot{\lambda} = -r\lambda$ so that the growth rate of consumption is

$$g_c = \theta^{-1}(r - \rho) \; . \tag{26}$$

This is equal to output growth on the balanced growth path. We can thus solve for the interest rate and obtain $r = \theta g + \rho$. The free entry condition for the technology firms imposes that profits from investing in patents must be zero. Revenue per unit of R&D investment is given by $V_Z$, cost is equal to $\frac{1}{\eta_Z}$. Consequently, we obtain $\eta_Z V_Z = 1$. Making use of equation (27), we obtain $\frac{\eta_Z \beta p_Z^{\frac{1}{\beta}} L}{r} = 1$. Solving this for $r$ and substituting it into equation (26) we obtain the following proposition:

$$g = \theta^{-1}(\beta \eta_Z L p_Z^{\frac{1}{\beta}} - \rho) \; .$$

Adding the extractive sector to the standard model by Acemoglu (2002), changes the interest part of the Euler equation, $g = \theta^{-1}(r - \rho)$.[18] Instead of two exogenous production factors, the interest rate $r$ in our model only includes labor, but adds the resource price, as $p_Z$ depends on $p_R$ according to equation (30). Together with (19), this yields the growth rate on the balanced growth path.

**Proposition 6** *Suppose that*

[18] There is no capital in this model, but agents delay consumption by investing in R&D as a function of the interest rate.

$$\beta\left[(1-\gamma)_R^{\varepsilon}(\eta_R\dot{R})^{\sigma-1}+\gamma_Z^{\varepsilon}(\eta_Z L)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}>\rho,\ \textit{and}$$

$$(1-\theta)\beta\left[\gamma_R^{\varepsilon}(\eta_R\dot{R})^{\sigma-1}+\gamma_Z^{\varepsilon}(\eta_Z L)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}<\rho.$$

*If* $(1-\gamma)^{\varepsilon}(\eta_R\delta\mu)^{1-\varepsilon}<1$ *the economy cannot produce. Otherwise, there exists a unique BGP equilibrium in which the relative technologies are given by equation* (32)*, and consumption and output grow at the rate in equation* (20)*.*[19]

[19] Starting with any $N_R(0)>0$ and $N_Z(0)>0$, there exists a unique equilibrium path. If $N_R(0)/N_Z(0)<(N_R/N_Z)^*$ as given by (32), then $M_{Rt}>0$ and $M_{Zt}=0$ until $N_{Rt}/N_{Zt}=(N_R/N_Z)^*$. If $N_R(0)/N_Z(0)>(N_R/N_Z)^*$, then $M_{Rt}=0$ and $M_{Zt}>0$ until $N_{Rt}/N_{Zt}=(N_R/N_Z)^*$. It can also be verified that there are simple transitional dynamics in this economy whereby starting with technology levels $N_R(0)$ and $N_Z(0)$, there always exists a unique equilibrium path, and it involves the economy monotonically converging to the BGP equilibrium of (20) like in Acemoglu (2002).

# Appendix 2 Directed Technological Change

Let $V_Z$ and $V_R$ be the BGP net present discounted values of new innovations in the two sectors. Then the Hamilton-Jacobi-Bellman Equation version of the value function for the intermediate good sector $r_t V_Z(j) - \dot{V}_Z(j) = \pi_Z(j)$ and the free entry condition of extraction technology firms imply that

$$V_Z = \frac{\beta p_Z^{1/\beta} L}{r^*}, \text{ and } V_R = \chi_R(j) - \psi_R \ , \tag{27}$$

where $r^*$ is the BGP interest rate, while $p_Z$ is the BGP price of the intermediate good and $\chi_R(j)$ is the BGP machine price in the extractive sector.

The greater is $V_R$ relative to relative to $V_Z$, the greater are the incentives to develop machines in the extractive sector rather than developing machines in the intermediate good sector. Taking the ratio of the two equations in (27) and including the equilibrium machine price (18) yields

$$\frac{V_R}{V_Z} = \frac{\chi_R(j) - \psi_R}{\frac{1}{r}\beta p_Z^{\frac{1}{\beta}} L} = \frac{\frac{1}{\eta_R}}{\frac{1}{r}\beta p_Z^{\frac{1}{\beta}} L} \ . \tag{28}$$

This expression highlights the effects on the direction of technological change

1. The price effect manifests itself because $V_R/V_Z$ is decreasing in $p_Z$. The greater is the intermediate good price, the smaller is $V_R/V_Z$ and thus the greater are the incentives to invent technology complementing labor. Since goods produced by

the relatively scarce factor are relatively more expensive, the price effect favors technologies complementing the scarce factor. The resource price $p_R$ does not affect $V_R/V_Z$ due to perfect competition among extraction technology firms and a flat supply curve.

2. The market size effect is a consequence of the fact that $V_R/V_Z$ is decreasing in $L$. Consequently an increase in the supply of labor translates into a greater market for the technology complementing labor. The market size effect in the intermediate good sector is defined by the exogenous factor labor. There is no equivalent in the extractive sector.

3. Finally, the cost of developing one new machine variety in terms of final output also influences the direction of technological change. If the parameter $\eta$ increases, the cost goes down, the relative profitability $V_R/V_Z$ decreases, and therefore the incentive to invent extraction technology declines.

Since the intermediate good price is endogenous, combining (26) with (28) the relative profitability of the technologies becomes

$$\frac{V_R}{V_Z} = \frac{\frac{1}{\eta_R}}{\frac{1}{r}\beta \left( p_R \frac{\gamma}{1-\gamma} \left( \frac{\delta \mu \dot{N}_R}{\frac{1}{1-\beta} p_Z^{\frac{1-\beta}{\beta}} N_Z L} \right)^{\frac{1}{\varepsilon}} \right)^{\frac{1}{\beta}} L} \tag{29}$$

Rearranging equation (9) we obtain

$$p_Z = \left( \gamma^{-\varepsilon} - \left( \frac{1-\gamma}{\gamma} \right)^{\varepsilon} p_R^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} . \tag{30}$$

Combining (30) and (19), we can eliminate relative prices, and the relative profitability of technologies becomes:

$$\frac{V_R}{V_Z} = \frac{\frac{1}{\eta_R}}{\frac{1}{r}\beta\left(\left(\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\left(\frac{1}{\eta_R} + \psi_R\right)\frac{1}{\mu\delta}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}\right)^{\frac{1}{\beta}} L}.$$

Using the free-entry conditions and assuming that both of them hold as equalities, we obtain the following BGP *technology market clearing* condition:

$$\eta_Z V_Z = \eta_R V_R. \tag{31}$$

Combining 31 with 29, we obtain the following BGP ratio of relative technologies and solving for $\frac{\dot{N}_R}{N_Z}$ yields:

$$\left(\frac{\dot{N}_R}{N_Z}\right)^* = \left(\left(\frac{r}{\eta_Z \beta L}\right)^{\beta} \frac{1-\gamma}{\gamma p_R}\right)^{\varepsilon} \frac{L p_Z^{\frac{1-\beta}{\beta}}}{(1-\beta)\delta\mu}$$

where the asterisk ($*$) denotes that this expression refers to the BGP value. The relative productivities are determined by both prices and the supply of labor.

## Appendix 3 Tables and Figures

| | | Aluminum | Copper | Lead | Tin | Zinc | Crude Oil |
|---|---|---|---|---|---|---|---|
| Range | | 1905-2009 | 1792-2009 | 1792-2009 | 1792-2009 | 1824-2009 | 1862-2009 |
| Constant | Coeff. | -1.774 | 0.572 | 0.150 | 1.800 | 1.072 | 8.242 |
| | t-stat. | (-0.180) | (0.203) | (0.052) | (0.660) | (0.205) | (0.828) |
| Lin.Trend | Coeff. | 0.008 | 0.009 | 0.016 | 0.001 | 0.014 | -0.021 |
| | t-stat. | (0.137) | (0.428) | (0.714) | (0.069) | (0.357) | (-0.317) |
| Range | | 1905-2009 | 1850-2009 | 1850-2009 | 1862-2009 | 1850-2009 | 1850-2009 |
| Constant | Coeff. | -1.299 | 0.109 | -0.268 | 2.439 | 1.894 | 7.002 |
| | t-stat. | (-0.200) | (0.030) | (-0.073) | (0.711) | (0.407) | (1.112) |
| Lin.Trend | Coeff. | 0.008 | 0.020 | 0.030 | -0.004 | 0.013 | -0.021 |
| | t-stat. | (0.137) | (0.518) | (0.755) | (-0.109) | (0.267) | (-0.317) |
| Range | | 1900-2009 | 1900-2009 | 1900-2009 | 1900-2009 | 1900-2009 | 1900-2009 |
| Constant | Coeff. | -0.903 | -1.428 | -0.490 | 1.068 | 2.764 | -1.974 |
| | t-stat. | (-0.239) | (-0.332) | (-0.102) | (0.269) | (0.443) | (-0.338) |
| Lin.Trend | Coeff. | 0.008 | 0.055 | 0.054 | 0.010 | 0.010 | 0.100 |
| | t-stat. | (0.137) | (0.820) | (0.713) | (0.168) | (0.099) | (1.106) |
| Range | | 1950-2009 | 1950-2009 | 1950-2009 | 1950-2009 | 1950-2009 | 1950-2009 |
| Constant | Coeff. | 2.269 | 1.556 | -3.688 | -0.061 | -0.515 | 3.445 |
| | t-stat. | (0.479) | (0.240) | (-0.505) | (-0.011) | (-0.062) | (0.354) |
| Lin.Trend | Coeff. | -0.055 | 0.041 | 0.198 | 0.049 | 0.103 | 0.090 |
| | t-stat. | (-0.411) | (0.225) | (0.958) | (0.307) | (0.441) | (0.326) |
| Range | | 1875-1975 | 1875-1975 | 1875-1975 | 1875-1975 | 1875-1975 | 1875-1975 |
| Constant | Coeff. | -0.549 | 1.323 | 0.370 | 3.719 | 1.136 | -1.111 |
| | t-stat. | (-0.088) | (0.266) | (0.081) | (0.812) | (0.176) | (-0.176) |
| Lin.Trend | Coeff. | -0.003 | 0.011 | 0.030 | -0.012 | 0.051 | 0.094 |
| | t-stat. | (-0.033) | (0.135) | (0.383) | (-0.152) | (0.468) | (0.875) |

Notes: The table presents coefficients and $t$-statistics for regressions of the growth rates on a constant and a linear trend.***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

Table 2: Tests of the stylized fact that the growth rates of real prices of mineral commodities equal zero and do hence not follow a statistically significant trend.

| | | Aluminum | Copper | Lead | Tin | Zinc | Crude Oil | World GDP |
|---|---|---|---|---|---|---|---|---|
| Range | | 1855-2009 | 1821-2009 | 1802-2009 | 1792-2009 | 1821-2009 | 1861-2009 | 1792-2009 |
| Constant | Coeff. | 48.464 | 4.86 | 16.045 | 4.552 | 30.801 | 35.734 | 0.128 |
| | t-stat. | *** 3.810 | *** 2.694 | *** 3.275 | * 2.231 | ** 2.58 | *** 4.365 | 0.959 |
| Lin.Trend | Coeff. | -0.221 | -0.006 | -0.087 | -0.016 | -0.174 | -0.182 | 0.018 |
| | t-stat. | ** -2.568 | -0.439 | ** -2.294 | -0.999 | * -1.975 | *** -3.334 | *** 16.583 |
| | | | | | | | | |
| Range | | 1855-2009 | 1850-2009 | 1850-2009 | 1850-2009 | 1850-2009 | 1861-2009 | 1850-2009 |
| Constant | Coeff. | 48.464 | 5.801 | 6.032 | 3.569 | 5.579 | 25.198 | 0.995 |
| | t-stat. | *** 3.810 | *** 3.461 | ***3.371 | * 2.185 | *** 3.774 | *** 4.81 | *** 5.49 |
| Lin.Trend | Coeff. | -0.221 | -0.018 | -0.038 | -0.015 | -0.021 | -0.182 | 0.019 |
| | t-stat. | ** -2.568 | -1.007 | -1.938 | -0.833 | -1.308 | *** -3.334 | *** 9.797 |
| | | | | | | | | |
| Range | | 1900-2009 | 1900-2009 | 1900-2009 | 1900-2009 | 1900-2009 | 1900-2009 | 1900-2009 |
| Constant | Coeff. | 19.703 | 5.965 | 2.980 | 2.844 | 4.44 | 9.883 | 2.004 |
| | t-stat. | *** 5.498 | *** 2.651 | * 2.043 | 1.361 | * 2.225 | *** 6.912 | *** 7.8 |
| Trend | Coeff. | -0.178 | 0.035 | -0.019 | -0.015 | -0.018 | -0.083 | 0.018 |
| | t-stat. | *** 3.174 | -0.995 | -0.853 | -0.464 | -0.592 | ***-3.711 | ***4.549 |
| | | | | | | | | |
| Range | | 1950-2009 | 1950-2009 | 1950-2009 | 1950-2009 | 1950-2009 | 1950-2009 | 1950-2009 |
| Constant | Coeff. | 10.781 | 5.043 | 13.205 | 0.051 | 5.675 | 9.897 | 4.729 |
| | t-stat. | *** 7.169 | *** 4.979 | *** 2.936 | 0.028 | *** 4.619 | *** 9.574 | *** 12.89 |
| Lin.Trend | Coeff. | -0.171 | -0.057 | -0.48 | 0.04 | -0.078 | -0.196 | -0.028 |
| | t-stat. | *** -3.999 | -1.978 | -1.553 | 0.768 | * -2.255 | *** -6.64 | *** -2.724 |
| | | | | | | | | |
| Range | | 1875-1975 | 1875-1975 | 1875-1975 | 1875-1975 | 1875-1975 | 1875-1975 | 1875-1975 |
| Constant | Coeff. | 50.75 | 6.307 | 3.851 | 3.762 | 4.384 | 12.272 | 1.244 |
| | t-stat. | *** 4.846 | ** 2.543 | 1.938 | 1.664 | * 2.032 | *** 4.060 | *** 5.509 |
| Lin.Trend | Coeff. | -0.53 | -0.024 | -0.018 | -0.026 | -0.005 | -0.072 | 0.027 |
| | t-stat. | *** -2.974 | -0.566 | -0.536 | -0.66 | -1.26 | -1.403 | ***7.045 |

Notes: The table presents coefficients and $t$-statistics for regressions of the growth rates on a constant and a linear trend. ***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

Table 3: Tests for the stylized facts that growth rates of world primary production and world real GDP are equal to zero and trendless. As our model does not include population growth, we run the same tests for the per capita data as a robustness check. The results are roughly in line with the results described above. See table 4 on the next page.

| | | Aluminum | Copper | Lead | Tin | Zinc | Crude Oil | World GDP |
|---|---|---|---|---|---|---|---|---|
| Range | | 1855-2009 | 1821-2009 | 1802-2009 | 1792-2009 | 1821-2009 | 1861-2009 | 1792-2009 |
| Constant | Coeff. | 48.301 | 5.474 | 20.57 | 4.427 | 30.7 | 35.689 | 0.032 |
| | t-stat. | *** 3.824 | *** 3.06 | *** 3.845 | * 2.181 | ** 2.584 | *** 4.379 | 0.276 |
| Lin.Trend | Coeff. | -0.229 | -0.018 | -0.125 | -0.023 | -0.182 | -0.19 | 0.01 |
| | t-stat. | *** -2.677 | -1.367 | *** -3.025 | -1.457 | * -2.071 | *** -3.499 | *** 11.066 |
| Range | | 1855-2009 | 1850-2009 | 1850-2009 | 1850-2009 | 1850-2009 | 1861-2009 | 1850-2009 |
| Constant | Coeff. | 48.301 | 5.399 | 5.629 | 3.179 | 5.18 | 24.681 | 0.628 |
| | t-stat. | *** 3.824 | *** 3.254 | ***3.169 | 1.961 | *** 3.541 | *** 4.733 | *** 4.052 |
| Lin.Trend | Coeff. | -0.229 | -0.027 | -0.047 | -0.024 | -0.03 | -0.19 | 0.01 |
| | t-stat. | *** -2.677 | -1.523 | ** -2.442 | -1.348 | -1.895 | *** -3.499 | *** 5.876 |
| Range | | 1900-2009 | 1900-2009 | 1900-2009 | 1900-2009 | 1900-2009 | 1900-2009 | 1900-2009 |
| Constant | Coeff. | 18.595 | 4.985 | 2.028 | 1.903 | 3.473 | 8.869 | 1.071 |
| | t-stat. | *** 5.242 | * 2.241 | 1.41 | 0.918 | 1.763 | *** 6.306 | *** 4.862 |
| Trend | Coeff. | -0.184 | -0.042 | -0.027 | -0.023 | -0.026 | -0.09 | 0.01 |
| | t-stat. | *** -3.315 | -1.214 | -1.186 | -0.694 | -0.404 | *** -4.084 | *** 3.01 |
| Range | | 1950-2009 | 1950-2009 | 1950-2009 | 1950-2009 | 1950-2009 | 1950-2009 | 1950-2009 |
| Constant | Coeff. | 8.583 | 2.952 | 1.141 | -1.954 | 3.578 | 7.716 | 2.632 |
| | t-stat. | *** 5.742 | *** 2.892 | 1.04 | 1.086 | *** 2.87 | *** 7.493 | *** 7.444 |
| Lin.Trend | Coeff. | -0.156 | -0.044 | -0.35 | 0.051 | -0.065 | -0.18 | -0.016 |
| | t-stat. | *** -3.667 | -1.515 | -1.129 | 0.997 | -1.819 | *** -6.14 | -1.551 |
| Range | | 1875-1975 | 1875-1975 | 1875-1975 | 1875-1975 | 1875-1975 | 1875-1975 | 1875-1975 |
| Constant | Coeff. | 50.004 | 5.854 | 3.413 | 3.317 | 3.942 | 11.789 | 0.834 |
| | t-stat. | *** 4.81 | ** 2.386 | 1.738 | 1.480 | 1.851 | *** 3.933 | *** 4.509 |
| Lin.Trend | Coeff. | -0.542 | -0.038 | -0.032 | -0.039 | -0.019 | -0.086 | 0.013 |
| | t-stat. | *** -3.06 | -0.908 | -0.959 | -1.028 | -0.517 | -1.691 | ***4.004 |

Notes: The table presents coefficients and $t$-statistics for regressions of the growth rates on a constant and a linear trend. ***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

Table 4: Tests for the stylized fact that growth rates of world per capita primary production and world per capita real GDP are equal to zero and trendless.

| | **Reserves/ Annual production** (Years) | **Crustal abundance/ Annual production** (Years) |
|---|---:|---:|
| Aluminum | 65[1ah] | 838[bch] |
| Copper | 30[ag] | 718[abg] |
| Iron | 44[ah] | 744[abh] |
| Lead | 18[ah] | 1,907[abh] |
| Tin | 18[ah] | 3,588[abh] |
| Zinc | 17[ah] | 842[abh] |
| Gold | 18[dh] | 2,170[efh] |
| Coal[2] | 65[gk] | |
| Crude oil[3] | 46[gk] | 729[6j] |
| Natural gas[4] | 41[gk] | |

Notes: We have used the following average annual growth rates of production from 1990 to 2010: Aluminum: 2.5%, Iron: 2.3%, Copper: 2%, Lead: 0.7%, Tin: 0.4%, Zinc: 1.6%, Gold: 0.6%, Crude oil: 0.7%, Natural gas: 1.7%, Coal: 1.9%, Hydrocarbons: 1.4%. [1] data for bauxite, [2] includes lignite and hard coal, [3] includes conventional and unconventional oil, [4] includes conventional and unconventional gas, [5] all organic carbon in the earth's crust. Sources: [a]U.S. Geological Survey (2012b), [b]Perman et al. (2003), [c]U.S. Geological Survey (2011c),[d]U.S. Geological Survey (2011b),[e]Nordhaus (1974),[f]U.S. Geological Survey (2010), [g]Federal Institute for Geosciences and Natural Resources (2011), [h]U.S. Geological Survey (2012a), [i]U.S. Bureau of Mines (1991), [j]Littke and Welte (1992), [k]British Petroleum (2013).

Table 5: Availability of selected non-renewable resources in years of production left in the reserve and crustal mass based on an exponentially increasing annual mine production (based on the average growth rate over the last 20 years).

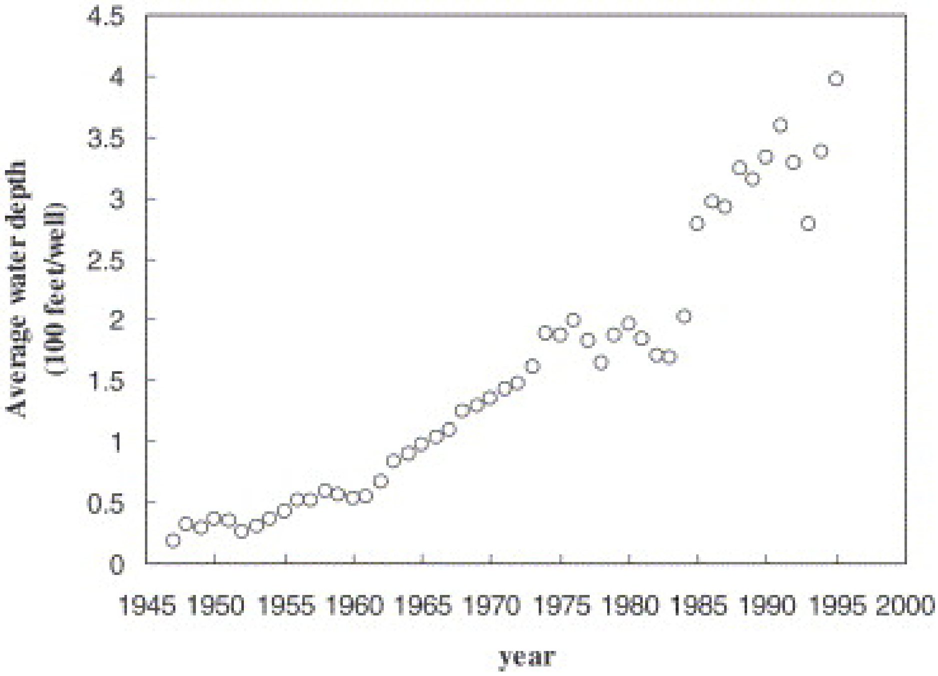


Figure 10: Average water depth of wells drilled in the Gulf of Mexico. Source: Managi et al. (2004).

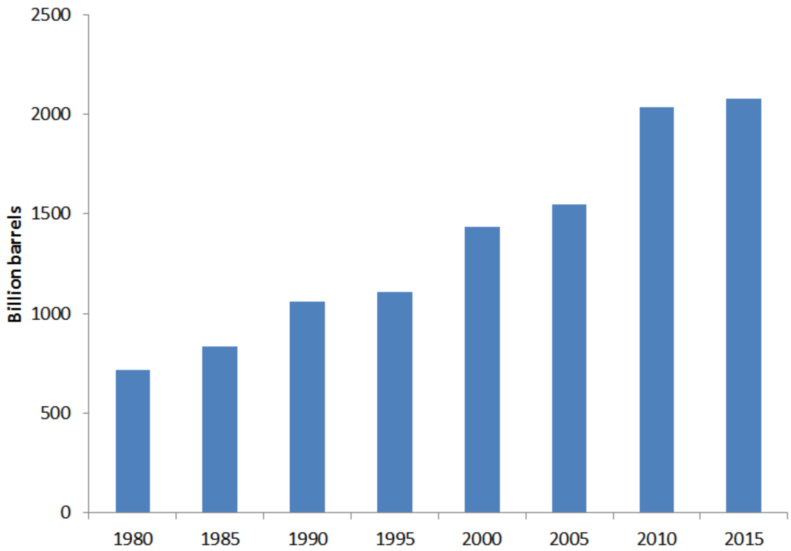


Figure 11: Historical evolution of oil reserves, including Canadian oil sands from 1980 to 2015. Source: BP, 2017.